Chapter 11: Other Logical Tools

Syllogisms and Quantification

Introduction

A persistent theme of this book has been the interpretation of logic as a set of practical tools. I have encouraged you to see these tools as beneficial in many contexts for keeping us on track for arriving at an objective, unforced agreement by encouraging us to test our beliefs. A very important implication of this interpretation is that as with any set of tools, the application of logical principles has contextual limitations. A hammer is a good tool for pounding nails, but surely unsuitable for the more delicate task of fixing the electronic components of a computer. Thus, the logical principles in Chapters 9 and 10 that we have distilled from our common sense should not be seen as absolute rules that guide our thinking in all contexts.

It is easy to show the limitations of the tools we have discussed thus far. Recall how the rule of commutation fails in certain contexts. If we are restricted to only the syntactical relations of propositional logic, we are forced to interpret "Mary became pregnant and married John" as equivalent to "Mary married John and became pregnant," $(P \bullet M) \equiv (M \bullet P)$. As we noted previously, propositional logic ignores the different meanings these statements would have in different cultural contexts.

Recall also the paradox of material implication discussed in Chapter 8. Our background knowledge about how the world works conflicts with some results in propositional logic. The statement, "If I sneeze within the next five minutes, then the whole world will disappear" is true, when the antecedent is false—I don't sneeze within the next five minutes.

In Chapter 8 it was also noted that although there will be limitations in capturing the rich texture of human experience, logicians have developed additional logics to capture other aspects of our common sense intuitions and explore the nature of rationality. Consider how propositional logic fails with the following argument.

11-1

> Either the accused had no <u>knowledge</u> that the crime took place or he is guilty of being an <u>accessory</u> after the fact. However, it is not true that if the accused was not in the <u>neighborhood</u> on the night of the crime, then he had no knowledge that the crime took

place. Thus it follows that the accused is guilty of being an accessory after the fact. (K, A, N)

We are surprised in reading the conclusion, for it obviously does not follow that from the mere possibility of someone knowing about a crime that the person was an accessory to the crime. Yet using the symbolic tools of propositional logic we can show that this argument is valid!

Translation:

1. ~K v A
2. ~(~N ⊃ ~K) /∴ A

Proof:

1. ~K v A	
2. ~(~N ⊃ ~K) /∴ A	
3. ~(N v ~K)	(2) Impl.
4. ~N • ~~K	(3) De M.
5. ~~K	(4) Com + Simp.
6. A	(1)(5) DS

The discrepancy between our intuition and the propositional proof is due to what logicians call the ***modal interpretation*** that is implied in the second premise. From the statement, "It is not true that if the accused was not in the neighborhood on the night of the crime, then he had no knowledge that the crime took place," ~(~N ⊃ ~K), it follows only that it is ***possible*** that the accused was not in the neighborhood but still knew about the crime. Not that it is ***necessary*** that the accused was not in the neighborhood but knew about the crime, ~N • K (line 4 above, plus DN). So, if we only know that it is possible that the accused knew about the crime, it does not follow that he was an accessory given premise 1.

A similar mismatch occurs between the statements, "It is not true that if John passes the final he will pass the course," ~(F ⊃ C), and "John passed the final, but he will not pass the course," F • ~C. Even though the second statement can be derived from the first: ~(F ⊃ C) ⇒ (implies) ~(~~F ⊃ C) ⇒ ~(~F v C) ⇒ (~~F • ~C) ⇒ (F • ~C) (DN + Impl. + De M. + DN). The problem again is that from "It is not true that if John passes the final he will pass the course," it should follow only that it is ***possible*** for John to pass the final and still not pass the course.

Because of such important nuances, logicians have developed a higher logic called ***modal logic*** to handle inferences that involve statements regarding necessity and possibility. In this higher logic, the inferential relationship between a negation of a conditional statement and a conjunction would look like this: ~(p ⊃ q) ≡ ◊(p • ~q). The diamond symbol as in ◊A is read as "It is possible that A is true." Another symbol, □, is also introduced, such that □A reads, "A is necessarily true." Various definitions are then given, such as

$\Diamond A$ = def. $\sim\Box\sim A$ (That "A is possibly true" is equivalent by definition to "It is not true that A is necessarily false," or "It is not true that A is impossible"),

and

$\Box A$ = def. $\sim\Diamond\sim A$ (That "A is necessarily true" is equivalent to "It is not true that not A is possible").

From these definitions, various rules are derived such that the chain of reasoning from step 2 to 4 in the above proof is rendered invalid.

There are also logics that deal with obligation and permissibility (called deontic logic), time (tense logic), multivalued logic which differs from standard logic in considering degrees of truth and shades of gray in place of the black and white crisp values of complete truth and falsity, and even quantum logic where key features of our common sense are violated.[1] These alternate logics are the subject of advanced courses, ranging from upper division to high level graduate courses. For the most part, however, they all use propositional logic as a basis or point of departure in the sense of either adding rules or showing contextual restrictions of propositional inferences.

Syllogisms and Quantification Logic

One group of additional logical tools that is often covered at the introductory level is called ***quantification logic*** or sometimes ***predicate logic***. Recall number 6 of the part III exercises in Chapter 1. To give a correct and thorough analysis of this exercise, essentially you had to be able to distinguish between the following two arguments:

11-2

No U.S. manufactured car built before 1970 was equipped with safety belts at the factory.
John has a U.S. manufactured car built in 1969.
Therefore, John's car was not equipped with safety belts at the factory.

[1] The legitimacy of these logics is controversial and is discussed in professional journals and philosophy books. In this context it is worth noting that the philosopher Robert C. Solomon has argued that it is a major philosophical mistake to equate our emotions with the irrational, that our emotions have a special logic of their own. See his ***The Passions*** (Garden City, New York: Anchor Press/Doubleday, 1976).

11-3

All U.S. manufactured cars built after 1970 were equipped with safety belts at the factory.
John has a U.S. manufactured car built in 1969.
Therefore, John's car was not equipped with safety belts at the factory.

The first argument (**11-2**) is valid, but the second (**11-3)** is invalid, committing a variation of the fallacy of denying the antecedent. However, we cannot show this symbolically using propositional logic alone. The techniques of propositional logic allow us to analyze simple statements and their compounds, but the validity and invalidity of the above arguments depend on the inner logical structure of noncompound statements and the meaning of the generalizations contained in the first premise of each argument. Thus, we need a way of symbolically picturing the difference between statements such as "All U.S. manufactured cars built after 1970 were equipped with safety belts at the factory," and "No U.S. manufactured car built before 1970 was equipped with safety belts at the factory."

Historically, a method for dealing with many noncompound statements was actually developed before modern propositional logic. The philosopher Aristotle (384-322 B.C.) is credited with being the first person to attempt to distill patterns of valid and invalid inference from our everyday arguing and judging. Called classical deduction theory, Aristotelian logic focuses on what are called ***categorical propositions*** and ***categorical syllogisms***. Categorical propositions make statements about the relationship of categories or classes of things. The classes are designated by ***terms***. In this logic, a syllogism is a very restricted deductive argument consisting of exactly three categorical propositions, a conclusion inferred from two premises, and the categorical propositions contain in total only three terms. For instance,

11-4

No U.S. manufactured car built before 1970 was equipped with safety belts at the factory.
Some U.S. manufactured Mustangs on the road today have safety belts equipped at the factory.
Therefore, some U.S. manufactured Mustangs on the road today were not built before 1970.

11-5

No U.S. manufactured car built before 1970 was equipped with safety belts at the factory.
All U.S manufactured Mustangs on the road today have safety belts equipped at the factory.
Therefore, no U.S manufactured Mustangs on the road today were built before 1970.

are syllogisms containing the terms (categories or classes):

U.S. manufactured cars built before 1970,

Cars equipped with safety belts at the factory,

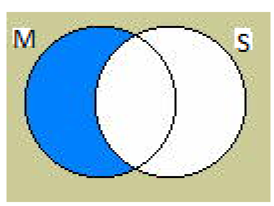
U.S manufactured Mustangs on the road today.

Notice also that these two syllogisms contain four types of propositions, traditionally referred to as ***universal affirmative***, ***universal negative***, ***particular affirmative***, and ***particular negative***, and labelled **A**, **E**, **I**, and **O**[2]:

A	(universal affirmative)	All U.S manufactured Mustangs on the road today have safety belts equipped at the factory.
E	(universal negative)	No U.S. manufactured car built before 1970 was equipped with safety belts at the factory.
I	(particular affirmative)	Some U.S manufactured Mustangs on the road today have safety belts equipped at the factory.
O	(particular negative)	Some U.S manufactured Mustangs on the road today were not built before 1970.

A traditional symbolic method for analyzing arguments of this type involves drawing pictures with overlapping circles, called ***Venn Diagrams***.[3] For instance, the basic claims of these four propositions can be represented as follows:

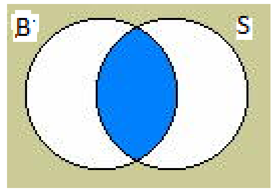
A **All M is S.** Everything in the M category of things is in the S category of things.



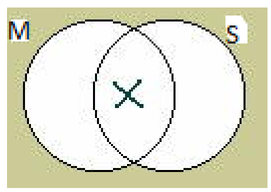
[2] The letter abbreviations most likely come from the Latin words "***A**ff**I**rmo*" (I affirm) and "*n**E**g**O***" (I deny).

[3] After the English mathematician and logician John Venn (1834-1923) who first used this method.

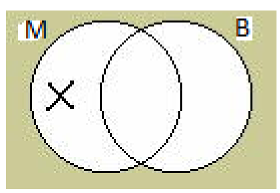
E **No B is S.** Anything that is in the B category of things is not in the S category of things.



I **Some M is S.** At least one thing that is in the M category of things is also in the S category of things.



O **Some M is not B.** At least one thing that is in the M category of things is not in the B category of things.

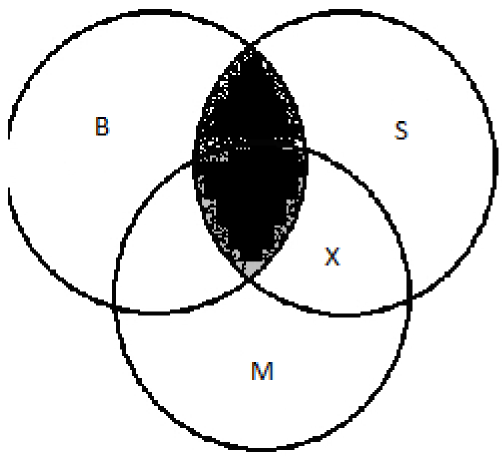


For the **A** proposition we draw two circles for each category (M = U.S manufactured **Mustangs** on the road today; S = Cars equipped with **Safety** belts at the factory), but shade that portion of the M circle that does not overlap with the S circle. In this way we picture the claim of the **A** proposition that everything that is in the M category of things is also in the S category of things. For the **E** proposition we shade the overlap area of the B (U.S. manufactured cars **built** before 1970) and S categories, indicating that there are no things that are in the B and S categories together. In both A and E propositions the shading indicates emptiness. For the **I** proposition, we put an X in the overlap area of the M and S categories, indicating that there is at least one thing that is both an M and an S. Finally, for the **O** proposition, we put an X in that portion of the M category that does not overlap with the B category, indicating that there is at least one thing that is an M, but is not a B.

We can now combine three circles, picturing the claims of each of the premises that make up the syllogisms **11-4** and **11-5**, and then view the result to see if the picture produced is consistent with the conclusion. If the picture constructed is consistent with the conclusion, then the argument is valid; if it is not, then the argument is invalid. Here is what Venn diagrams of **11-4** and **11-5** would look like:

11-4

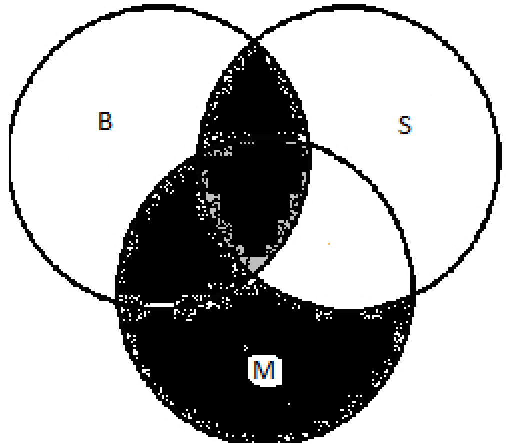
No U.S. manufactured car built before 1970 was equipped with safety belts at the factory.
Some U.S. manufactured Mustangs on the road today have safety belts equipped at the factory.
Therefore, some U.S. manufactured Mustangs on the road today were not built before 1970.



11-4 No B is S.
Some M is S.
Therefore, some M is not B.

11-5

No U.S. manufactured car built before 1970 was equipped with safety belts at the factory.
All U.S manufactured Mustangs on the road today have safety belts equipped at the factory.
Therefore, no U.S manufactured Mustangs on the road today were built before 1970.



11-5 No B is S.
All M is S.
Therefore, no M is B.

Notice that in **11-4** we first shade the overlapping area of B and S to picture the first premise that no B is S. Then we put an X in the overlapping area of the categories M and S to picture the claim of the second premise that at least one M is an S. Once this is done, we see that the picture we have created is consistent with the conclusion that at least one M is not a B as shown by the fact that there is an X in the M circle but not in the B circle. Thus, **11-4** is valid. If the conclusion stated that "Some U.S manufactured Mustangs on the road today were built before 1970," the argument would be invalid, because after drawing the pictures for the premises there is no X in the overlapping area of the B and M circles as would be required to picture the conclusion.

In **11-5** we again shade the overlapping area of the B and S circles to picture the claim of the first premise that no B is S. Then we shade all of the M circle that does not overlap with the S circle to picture the claim of the second premise that all M is S. This leaves us with a picture that is consistent with the conclusion, no M is B, because the overlap area of the M and B circles is shaded. So, **11-5** is valid. Notice that if the conclusion were "All U.S manufactured Mustangs on the road today were built before 1970," (All M is B.) then the argument would be invalid because there is no open unshaded area of the M circle contained in the B circle as would be required to picture this claim. When the conclusion states more than what is contained in the premises, deductive arguments are always invalid.

Venn diagrams are fun and can be used to capture and draw out numerous nuances of common sense reasoning. They can also be used to picture the many rules of syllogistic reasoning discovered by Aristotle and his followers in the Middle Ages who further developed classical logic. But they cannot be used to adequately analyze the arguments **11-2** and **11-3** above, or arguments that contain premises such as "In the 1990's the first electric vehicles were both expensive and inconvenient," or complicated statements such as the quotation from Lincoln at the beginning of the first chapter, "You can fool some of the people all of the time and all of the people some of the time, but you cannot fool all of the people all of the time." Thus, by adding a few commonsense rules to propositional logic, modern logicians have developed a more powerful set of tools called ***quantification logic*** that combines the distinctive elements of syllogistic logic and propositional logic.

First let's see how terms in syllogistic logic would be symbolically represented in quantification logic.

U.S. manufactured cars built before 1970.
    Bx = x is a U.S. manufactured car ***B***uilt before 1970.
Cars equipped with safety belts at the factory.
    Sx = x is a car equipped with ***S***afety belts at the factory.
U.S manufactured Mustangs on the road today.
    Mx = x is a U.S. manufactured ***M***ustang on the road today.

Like the **p**, **q**, and **r** in propositional logic, the letter **x** in quantification logic is a ***variable***. It does not represent a particular car built before 1970, a particular car equipped with safety belts, or a particular Mustang on the road today. Bx, Sx, and Mx are not propositions that are true or false; they are ***propositional functions***, empty places waiting, so to speak, for something to be substituted for the variables. In quantification logic, to represent individual cars, persons, and things ***individual constants*** are used and designated by the lower case letters ***a*** through ***w***. For example,

Bj = John's car was built before 1970.
Sj = John's car was equipped with safety belts at the factory.
Mj = John's car is a U.S. manufactured Mustang on the road today.

Bj, Sj, and Mj are ***substitution instances*** of Bx, Sx, and Mx. Just as the variable **p** in propositional logic could stand for any statement whatsoever, the propositional function **x** could stand for Annelise's car (Ba, Sa, Ma), or Cesar's car (Bc, Sc, Mc), or Tran's car (Bt, St, Mt), and so on.

Next, ***quantification symbols*** are introduced to capture the different meanings of generalizations (All, No) and, what we will now call existential statements (Some, Some not). Here is how our four basic categorical propositions would be translated in quantification logic:

A	(universal affirmative)	All U.S manufactured Mustangs on the road today have safety belts equipped at the factory.

All M is S = Given any x, if x is an M then x is also an S.

$(x)(Mx \supset Sx)$

E (universal negative) No U.S. manufactured car built before 1970 was equipped with safety belts at the factory.

No B is S = Given any x, if x is a B then x is not an S.

$(x)(Bx \supset \sim Sx)$

I (particular affirmative) Some U.S manufactured Mustangs on the road today have safety belts equipped at the factory.

Some M is S = There is at least one x such that x is M and x is S.

$(\exists x)(Mx \bullet Sx)$

O (particular negative) Some U.S manufactured Mustangs on the road today were not built before 1970.

Some M is not B = There is at least one x such that x is M and x is not B.

$(\exists x)(Mx \bullet \sim Bx)$

Note that (x) symbolizes "Given any x," and ($\exists$x) symbolizes "There is at least one x." We can now translate **11-2** and **11-3** as follows:

No U.S. manufactured car built before 1970 was equipped with safety belts at the factory.
John has a U.S. manufactured car built in 1969.
Therefore, John's car was not equipped with safety belts at the factory.

1. $(x)(Bx \supset \sim Sx)$
2. Bj[4] $/\therefore \sim Sj$

All U.S. manufactured cars built after 1970 were equipped with safety belts at the factory.
John has a U.S. manufactured car built in 1969.
Therefore, John's car was not equipped with safety belts at the factory.

1. $(x)(\sim Bx \supset Sx)$[5]

[4] Here we will interpret "a U.S. manufactured car built in 1969" as "a U.S. manufactured car built before 1970."

[5] Here we interpret "a U.S. manufactured car built (in or) after 1970" as "a U.S. manufactured car not built before

2. Bj /∴ ~Sj

With the addition of a few rules to capture the correct inferences involving (x) and (∃x), we can show symbolically that **11-2**, **11-4,** and **11-5** are valid arguments. Before doing so, however, let's create a dictionary of quantification statements similar to the one in Chapter 7 for propositional logic.

Usage Dictionary

1. All humans are mortal. (Given any x, if x is an H, then x is an M.)

   (x)(Hx ⊃ Mx)

2. Only registered voters are eligible to vote. (Given any x, if x is an E, then x is an R.)

   (x)(Ex ⊃ Rx)

3. All eligible voters must be registered to vote. (Given any x, if x is an E, then x is an R.)

   (x)(Ex ⊃ Rx)

4. Socrates is human. (s is H.)

   Hs

5. If Socrates is human, then Socrates is mortal. (If s is H, then s is M.)

   Hs ⊃ Ms

6. Everything is mortal. (Given any x, x is M.)

   (x)Mx

7. No man is an island. (Given any x, if x is an M, then x is not an I.)

   (x)(Mx ⊃ ~Ix)

8. Nothing is perfect. (Given any x, x is not P.)

1970," ~Bx.

$(x)\sim Px$

9. Dictators are not good leaders. (Given any x, if x is a D, then x is not a G.)

$(x)(Dx \supset \sim Gx)$

10. Some marriages are happy. (There is at least one x, such that x is M and x is H.)

$(\exists x)(Mx \bullet Hx)$

11. There are happy marriages. (There is at least one x, such that x is M and x is H.)

$(\exists x)(Mx \bullet Hx)$

12. Some marriages are not happy. (There is at least one x, such that x is M and x is not H.)

$(\exists x)(Mx \bullet \sim Hx)$

13. There are unhappy marriages. (There is at least one x, such that x is M and x is not H.)

$(\exists x)(Mx \bullet \sim Hx)$

14. Happiness exists. (There is at least one x, such that x is H.)

$(\exists x)Hx$

15. It is not true that some presidents of the U.S. were not rich. (It is not true that there is an x, such that x is P and x is not R. Given any x, if x is P, then x is R.)

$\sim(\exists x)(Px \bullet \sim Rx)$ or $(x)(Px \supset Rx)$

16. It is not the case that every smoker gets lung cancer. (It is not true that given any x, if x is an S, then x is an L. There is at least one x, such that x is an S and x is not an L.)

$\sim(x)(Sx \supset Lx)$ or $(\exists x)(Sx \bullet \sim Lx)$

17. Only Democrats and Republicans have a realistic opportunity for elective office in the U.S. (Given an x, if x is an O, then x must be either a D or a R.)

$(x)[Ox \supset (Dx \vee Rx)]$

18. Black holes exist if and only if no other explanations for what we observe regarding unusual binary systems and the center of galaxies are credible. (There is at least one x, such that x is a B if and only if given any x, if x is an E, then x is not C.)

$(\exists x)Bx \equiv (x)(Ex \supset \sim Cx)$

19. If some politicians lie, then some voters are cynical. (If there is at least one x, such that x is a P and an S, then there is at least one x, such that x is a V and x is a C.)

$(\exists x)(Px \bullet Lx) \supset (\exists x)(Vx \bullet Cx)$

20. In the late 1990's electric vehicles were both costly and inconvenient. (Given any x, if x was an E, then x was a C and x was an I.)

$(x)[Ex \supset (Cx \bullet Ix)]$

Dictionary Elaboration

As in the case of translations in propositional logic, the goal of translating in quantification logic is to do the best we can in capturing the meaning of the statements in ordinary language. Often we will find statements that do not have the exact grammatical form of our **A**, **E**, **I**, and **O**, propositions, but can be interpreted into their symbolic equivalents. For instance, number 1 in the dictionary could have been expressed in ordinary language as "Humans are mortal." Similarly, notice that number 9 is another way of saying "No dictators are good leaders," and numbers 10 and 12 are different ways of saying 11 and 13 respectively.

Recall how in Chapter 1 the different meaning between *all* and *only* easily led to mistakes in reasoning. An examination of numbers 2 and 3 reveals the secret behind translating *only* statements. As with *only if* statements in propositional logic, the term or category of things that follows the *only* will always be translated as a consequent. Thus, the secret to translating *only* statements is to always replace *only* with *all* and reverse the terms. So, the statement from Chapter 1 "Only people who believe in God are moral" becomes "All people who are moral believe in God," $(x)(Mx \supset Gx)$.

As with *only if* statements in propositional logic, *only* statements in quantification logic are interpreted as specifying a necessary condition. In numbers 2 and 3 in the dictionary, being registered to vote is specified as a necessary condition for being eligible to vote. Recall from Chapter 1 that the reason many people will object to the statement, "Only people who believe in God are moral," is that it claims that believing in the Judeo-Christian God is necessary to be a good person, which excludes Buddhists, atheists, agnostics, and secular humanists who believe that it is possible to be moral without divine aid.

Note also that there are other ways of saying *only*. The statement, "None but the brave deserve the fair," is the same as "Only the brave deserve the fair." Hence, it would be translated as, $(x)(Fx \supset Bx)$.

Numbers 15 and 16 show that negated existential statements can be interpreted as universal statements, and negated universal statements can be interpreted as existential statements. Consider the logic behind these interpretations. Stating "It is not true that something exists that is perfect," $\sim(\exists x)Px$, states the same thing as number 8, "Nothing is perfect," $(x)\sim Px$. So,

$$\sim(\exists x) \equiv (x)\sim$$

thus number 15, $\sim(\exists x)(Px \bullet \sim Rx)$, is equivalent to $(x)\sim(Px \bullet \sim Rx)$, and

1. $(x)\sim(Px \bullet \sim Rx)$	
2. $(x)(\sim Px \vee \sim\sim Rx)$	(1) De M.
3. $(x)(\sim Px \vee Rx)$	(2) DN
4. $(x)(\sim\sim Px \supset Rx)$	(3) Impl.
5. $(x)(Px \supset Rx)$	(4) DN

This proves symbolically that, "It is not true that some presidents of the United States were not rich," can be stated more simply as "All presidents of the United States were rich." Similar considerations apply to number 16, because $\sim(x) \equiv (\exists x)\sim$. Shortly, we will formalize these equivalencies and add them as rules in quantification logic.

Numbers 17-20 show that quantification logic can capture the meaning of statements that are much more complicated than simple categorical propositions. Number 17 also shows that translating is not a rigid mechanical process, that the meaning must be understood in ordinary language first. Even though the word *and* is used in number17, the meaning is clearly that it is necessary to be *either* a Democrat *or* a Republican to have a chance at elective office, not that one should be both.

Numbers 14 and 18 show that existence is never a term. We would not translate "Happiness exists" as $(\exists x)(Hx \bullet Ex)$, for the $(\exists x)$ symbol already specifies an existential claim. Otherwise, we would be committing ourselves to the very strange metaphysical position that there are categories of things that exist and categories of things that do not exist. Even though our language allows us to say, "Some things don't exist," it appears reasonable to believe that it is a conceptual mistake to conclude from this that there are actual kinds of things that don't exist. We would make the same mistake the Queen does in *Alice in Wonderland* when Alice tells her that "Nobody is coming down the roadway," and the Queen remarks how keen Alice's eyesight is to be able to see nobody. Similarly, I can say, "Two-headed dragons do not exist," but it seems unreasonable to interpret this as meaning that there is a category for things that do not exist and a two-headed dragon is one of the actual things in this category!

Such considerations are not mere ivory tower distinctions. Historically, the mistake of interpreting existence as a term can be seen as an attempted major reinforcement for an entire culture. The eighteenth century German philosopher and physicist Immanuel Kant showed that philosophers and theologians in the Middle Ages made this mistake when they argued that one could prove the existence of God by merely contemplating the definition of God as the most perfect being. Known as the *ontological argument* for the existence of God, they argued that because by definition the most perfect being had to exist, otherwise it would not be perfect, God must exist. To use quantification symbolism, they incorrectly assumed that given any x, if x is perfect, then x exists, $(x)(Px \supset Ex)$. Then they reasoned if God is perfect, then God must exist, $Pg \supset Eg$. Since by definition God is perfect, Pg, it follows (modus ponens) that God must exist, Eg. Viewed this way their argument was valid. We will see why shortly when a new rule is discussed, but for now understand that by modern standards this proof fails because the first premise should be translated as, "There is an x, such that x is perfect," $(\exists x)Px$. This statement makes a contingent existential claim that is true or false.

Kant's criticism of the ontological argument underscores the obvious point that we cannot deduce the existence of something from a definition of that something. I can define what it means to have \$100, but this does not mean that I have \$100 in my wallet.

Astronomers and physicists are now very confident that black holes do exist. Observations can be made of unusual binary systems—two stars orbiting each other—where one companion is not visible but is obviously very massive. The invisible companion is also a high energy X-ray source, an indication of matter being heated as it falls onto a very massive object. Furthermore, observations with the Hubble telescope show rapidly revolving disks of matter at the center of most galaxies. These disks are revolving much too swiftly to have a normal gravitational object at the center. Number 18 in the usage dictionary summarizes the inductive claim that it is reasonable to believe in black holes given that all other explanations for what we observe about these matters are unreasonable.

Number 18 also shows that when we translate we must assume or specify a ***domain of discourse***. In discussing explanations for what we observe related to black holes, we do not mean just any explanations for any observations in general, but a particular subset of observations, that is those related to the topic of black holes, what we observe regarding binary star systems and the center of galaxies. Recall that the arguments on Mustangs and seat belts specified the domain of U.S. manufactured cars. Consider this argument:

11-6

No U.S. manufactured car built before 1970 was equipped with safety belts at the factory.
All Hondas on the road today have safety belts equipped at the factory.

Therefore, no Honda on the road today was built before 1970.

Because Hondas have been manufactured in Japan and the United States, the validity of this argument cannot be established unless the domain of U.S.-manufactured cars is specified throughout the argument. The second premise is consistent with the possibility that some Hondas were built in another country before 1970 and equipped with safety belts at the factory. So, unless we specify that only U.S.-manufactured Hondas are being discussed in the second premise and the conclusion, both premises could be true and the conclusion false; it is possible that some Hondas still on the road today were equipped with safety belts in Japan before 1970. Similarly, if I tell my students, "Everyone passed the midterm," they know that I am referring to the domain of people in my course, not everyone in the world.

Number 19 shows that statements can propositionally combine quantifiers, and 20 shows that quantified propositions can have a large number of grammatical variations. The original statement for this translation could have been, "All electric vehicles during the late 1990's were both costly and inconvenient." As we have noted several times in this book, the English language is much richer than the symbolic pictures we devise. Thus, as in the case of propositional translations, there can be no mechanical procedure for translating quantified statements. In every case the meaning of the sentence, including the appropriate domain, must be understood first and then captured as best we can with logical connectives and quantifiers.

Exercises I: Translations

Translate each of the following using logical connectives and quantifiers. Use the abbreviations suggested.

1. All charitable donations are tax-deductible. (Cx = x is a charitable donation; Tx = x is tax-deductible.)

2. No council member voted for the bill. (Cx = x is a council member; Vx = x voted for the bill.)

3.* Every car parked on the street after 9:00 pm will be ticketed. (Cx = x is a car parked on the street after 9:00 pm; Tx = x is a car ticketed.)

4. Some 2010 U.S.-manufactured cars are dependable. (Ux = x is a 2010 U.S.-manufactured car; Dx = x is dependable.)

5. Japanese luxury cars are dependable. (Jx = x is a Japanese luxury car; Dx = x is dependable.)

6. Metals conduct electricity. (Mx = x is a metal; Ex = x conducts electricity.)

7. Some China stocks are not good investments. (Sx = x is a China stock; Gx = x is a good investment.)

8.* Not every politician makes decisions with reelection primarily in mind. (Px = x is a politician; Mx = x makes decisions with reelection primarily in mind.)

9. There are no stars in our galaxy that are not massive. (Sx = x is a star in our galaxy; Mx = x is massive.)

10. Every student who maintains a quiz average of over 90% can miss one exam. (Sx = x is a student; Qx = x maintains a quiz average of over 90%; Mx = x can miss one exam.)

11. All Democrats are either liberal or moderate. (Dx = x is a democrat; Lx = x is liberal; Mx = x is moderate.)

12. Only persons over 62 are eligible for Social Security benefits. (Ox = x is a person over 62; Sx = x is eligible for Social Security benefits.)

13.* Whales and humans are both mammals. (Wx = x is a whale; Hx = x is a human; Mx = x is a mammal.)

14. Quarks exist, but dragons do not exist. (Qx = x is a quark; Dx = x is a dragon.)

15. Cats are not the only animals in my house. (Cx = x is a cat; Ax = x is an animal in my house.)

16. Not every student applicant will be accepted into the medical school. (Sx = x is a student applicant; Ax = x will be accepted into the medical school.)

17.* Not only students but faculty as well attended the initiation dance. (Sx = x is a student; Fx = x is a faculty member; Ax = x attended the initiation dance.)

18. Those who ignore the history of Afghanistan are condemned to repeat it. (Ix = x ignores the history of Afghanistan; Cx = x is condemned to repeat the history of Afghanistan.)

19. Not all rocks that glitter contain gold. (Gx = x is a rock that glitters; Cx = x is a rock that contains gold.)

20. The truth is not always helpful, but deception is never helpful. (Tx = x is the truth; Hx =

x is helpful; interpret "deception" as "not the truth.")

Proving Validity in Quantification Logic

We are now ready to devise a method of formal proof for arguments in quantification logic. To do so, we must add to our list of 19 rules used in propositional logic. Consider **11-2** again:

> No U.S.-manufactured car built before 1970 was equipped with safety belts at the factory.
> John has a U.S.-manufactured car built in 1969.
> Therefore, John's car was not equipped with safety belts at the factory.

1. (x)(Bx ⊃ ~Sx)
2. Bj /∴ ~Sj

Even translated into the new notation of quantification logic, this argument looks suspiciously like modus ponens. What is needed prior to applying modus ponens is the intuitively obvious inference that if, given any x, if x is a B then x is not an S, then it follows that if j is a B, then j is not an S.

1. (x)(Bx ⊃ ~Sx)
   /∴ Bj ⊃ ~Sj

If all U.S.-manufactured cars built before 1970 did not have safety belts equipped at the factory, then if John has a U.S.-manufactured car built before 1970, he must have a car that was not equipped with safety belts at the factory. In other words, if (x)(Bx ⊃ ~Sx) is true, then so is Ba ⊃ ~Sa, Bb ⊃ ~Sb, Bc ⊃ ~Sc, and so on. We can substitute many individual constants with the domain of U.S. manufactured cars. This is simply what it means to assert a generalization. If (x)(Bx ⊃ ~Sx) is true, then any substitution instance, Bj ⊃ ~Sj, of this proposition is true.

In quantification logic this inference is called ***Universal Instantiation*** (**UI**), because we are inferring "instances" from a universal statement, and it becomes our first new rule added to the 19 rules of propositional logic. We can summarize this rule as follows:

UI (x)(Ψ**x**)
/∴ Ψυ

The Greek ***psi*** symbol (Ψ) is a variable representing an empty place for any terms whatsoever of a given domain, and the Greek ***nu*** symbol (υ) is also a variable representing any substituted individuals whatsoever in that domain.

Using this rule we can now provide a simple proof for **11-2**.

1. $(x)(Bx \supset \sim Sx)$
2. Bj $/\therefore$ $\sim Sj$
3. $Bj \supset \sim Sj$ (1) UI
4. $\sim Sj$ (2)(3) MP

Next, let's consider **11-5** again with its translation:

> No U.S.-manufactured car built before 1970 was equipped with safety belts at the factory.
> All U.S.-manufactured Mustangs on the road today have safety belts equipped at the factory.
> Therefore, no U.S.- manufactured Mustang on the road today was built before 1970.

1. $(x)(Bx \supset \sim Sx)$
2. $(x)(Mx \supset Sx)$ $/\therefore$ $(x)(Mx \supset \sim Bx)$

This argument looks suspiciously like a subroutine we covered in Chapter 10.

1. $p \supset q$
2. $r \supset \sim q$ $/\therefore$ $r \supset \sim p$
3. $\sim q \supset \sim p$ (1) Trans.
4. $r \supset \sim p$ (2)(3) HS

To use this subroutine, however, we need to employ some obvious valid inferences: (1) if all U.S.-manufactured cars built before 1970 did not have safety belts equipped at the factory, then *any arbitrarily selected individual* U.S.-manufactured car built before 1970 did not have safety belts equipped at the factory; (2) if *any arbitrarily selected individual* U.S.-manufactured Mustang on the road today was not built before 1970, then no U.S.-manufactured Mustang on the road today was built before 1970. Concerning (1), we see that our instantiation rule applies not only to individuals—**a, b, c,** and so on—but to any arbitrarily selected individual, which we will use the italicized *y* to designate. So from $(x)(Bx \supset \sim Sx)$, $By \supset \sim Sy$ follows, and from $(x)(Mx \supset Sx)$, $My \supset Sy$ follows, and from our subroutine, $My \supset \sim By$:

1. $(x)(Bx \supset \sim Sx)$
2. $(x)(Mx \supset Sx)$ $/\therefore$ $(x)(Mx \supset \sim Bx)$
3. $By \supset \sim Sy$ (1) UI

4. M*y* ⊃ S*y* (2) UI
5. ~~S*y* ⊃ ~B*y* (3) Trans.
6. S*y* ⊃ ~B*y* (5) DN
7. M*y* ⊃ ~B*y* (4)(6) HS

Finally, the second obvious intuitive inference shows that we can add a new rule to our developing quantification logic. We will call this new rule ***Universal Generalization*** (**UG**), and it is summarized as follows:

UG Ψ***y***
/∴ **(x)**(Ψx)

This rule indicates that we are entitled to generalize from the special instantiation *y*, which indicates any arbitrarily selected individual. We can now complete the proof of **11-5**.

1. (x)(Bx ⊃ ~Sx)
2. (x)(Mx ⊃ Sx) /∴ (x)(Mx ⊃ ~Bx)
3. B*y* ⊃ ~S*y* (1) UI
4. M*y* ⊃ S*y* (2) UI
5. ~~S*y* ⊃ ~B*y* (3) Trans.
6. S*y* ⊃ ~B*y* (5) DN
7. M*y* ⊃ ~B*y* (4)(6) HS
8. (x)(Mx ⊃ ~Bx) (7) UG

Notice that there is nothing magical about instantiating to and then generalizing from *y*, provided that we start with universal statements. This restriction must apply otherwise we would be endorsing the fallacy of hasty conclusion as a valid rule. Clearly from the proposition, "At least one Mustang on the road today does not have safety belts equipped at the factory," (∃x)(Mx • ~Sx), it does not follow that "All Mustangs on the road today do not have safety belts equipped at the factory," (x)(Mx ⊃ ~Sx). What prevents this invalid inference in quantification logic is that the special instantiation to *y*, designating any arbitrarily selected individual from a domain, is validly inferred only from a universal statement. Thus, the following is invalid and should never be used in a proof:

(∃x)(Ψ**x**)
/∴ Ψ***y*** (**incorrect**)

However, there are valid rules involving existential statements and we will need these rules to complete our augmentation of propositional logic. For instance, from the statement, "John has a U.S.-manufactured Mustang on the road today with safety belts equipped at the factory," Mj • Sj, it clearly follows that at least one U.S- manufactured Mustang on the road today has safety belts equipped at the factory, or "Some U.S-manufactured Mustangs on the road today have safety belts equipped at the factory," $(\exists x)(Mx \bullet Sx)$. We will call this rule ***Existential Generalization*** (**EG**) and summarize it as follows:

$$\textbf{EG} \qquad \begin{array}{l} \Psi\upsilon \\ /\therefore \ \ \boldsymbol{(\exists x)(\Psi x)} \end{array}$$

where Ψ represents any term or terms whatsoever, and υ represents any individual constant whatsoever. Note that in this case an inference from Ψy to $(\exists x)\Psi x$ would be valid, for it surely follows that if any arbitrarily selected individual has a certain property, then there is some thing that has that property.

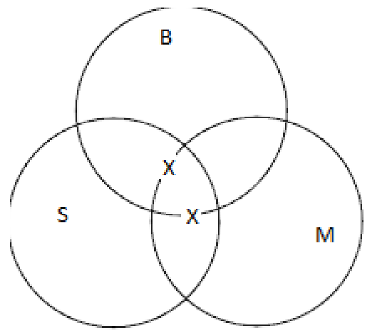
At first this may seem like a bad rule. Could we conclude from a general premise about mythical unicorns having sharp horns that unicorns actually exist? The answer is no, because although it follows from "All unicorns have sharp horns," $(x)(Ux \supset Sx)$, that any arbitrarily selected individual unicorn has a sharp horn, $Uy \supset Sy$, $Uy \bullet Sy$ cannot be derived unless we establish Uy, that an arbitrarily selected individual unicorn exists. So, if $Uy \bullet Sy$ cannot be derived from $(x)(Ux \supset Sx)$, then neither can $(\exists x)(Ux \bullet Sx)$.

There is also a version of instantiation that is valid for existential statements, but it must be used with great care. If some U.S-manufactured Mustangs on the road today have safety belts equipped at the factory, then it follows that there is at least one U.S.-manufactured Mustang somewhere on the road today that has safety belts equipped at the factory. However, in any given context we cannot assume that we know that this refers to any specific Mustang. We cannot say that this hypothetical Mustang is Mary's, Cesar's, Kanoe's, or Tran's. This realization prevents the following argument from being valid.

11-7

Some U.S-manufactured Mustangs on the road today have safety belts equipped at the factory.
Some cars built before 1970 had safety belts equipped at the factory.
Thus, some U.S-manufactured Mustangs on the road today were built before 1970.

Clearly, although the conclusion may be true, it goes beyond or infers more than what is implied in the premises. If some cars built before 1970 had safety belts equipped at the factory, and some U.S-manufactured Mustangs on the road today also have safety belts equipped at the factory, then it is possible that it was these U.S-manufactured Mustangs that were built before 1970. But there is no guarantee; we are not locked in to this conclusion. It is just as possible that all U.S-manufactured Mustangs on the road today were built after 1970 and equipped with safety belts at the factory, and that expensive Cadillacs were the first cars equipped with safety belts at the factory. The uncertainty of this conclusion, given the premises, can be shown by a Venn diagram.



11-7 Some M is S.
Some B is S.
Therefore, some M is B.

Notice that in mapping the first premise, we have a choice of where to put the X. We could put it in that portion of the overlap of the M and S circle that also overlaps with the B circle or just in that portion that overlaps S and M alone. The appropriate place to put it then is on the line that separates these two possibilities, thus indicating the undecidedness. A similar uncertainty results for the second premise, "Some B is S." We could put the X in that portion of overlap between just the B and S circles, or in that portion of the overlap of the B, S, and M circles. Thus, the appropriate place to put the X is on the line that separates these two possibilities. This leaves us with a Venn picture that does not explicitly contain the conclusion. Both X's are on the borders of the overlap section of the M and B circles, indicating that it is only possible that some M's are B's.

In quantification logic, to be consistent with this result we need to have the following important restriction when using the next quantification rule, which we will call ***Existential Instantiation***, (**EI**). In addition to not being able to instantiate to the special symbol *y*, we can instantiate existentially to an individual constant, ***provided that the individual constant occurs nowhere earlier in the context of the argument***. Here then is a summary of **EI**, followed by its correct use in showing **11-4** above to be valid and an incorrect use in attempting to show **11-7** to be valid.

EI **(∃x)(Ψx)**
/∴ Ψυ

Where υ is neither *y* nor any individual constant having a previous occurrence in the context of the argument.

11-4

1. (x)(Bx ⊃ ~Sx)	
2. (∃x)(Mx • Sx) /∴	(∃x)(Mx • ~Bx)
3. Ma • Sa	(2) EI
4. Ba ⊃ ~Sa	(1) UI
5. Ma	(3) Simp.
6. Sa	(3) Com. + Simp.
7. ~~Sa	(6) DN
8. ~Ba	(4)(7) MT
9. Ma • ~Ba	(5)(8) Conj.
10. (∃x)(Mx • ~Bx)	(9) EG

11-7

1. (∃x)(Mx • Sx)	
2. (∃x)(Bx • Sx) /∴	(∃x)(Mx • Bx)
3. Ma • Sa	(1) EI
4. Ba • Sa	(2) EI (**incorrect !!**)
5. Ma	(3) Simp.
6. Ba	(4) Simp.
7. Ma • Ba	(5)(6) Conj.
8. (∃x)(Mx • Bx)	(7) EG

Note that the restriction applies to the conclusion of an argument as well. In saying that the individual constant cannot occur anywhere earlier in the context of an argument, this includes any constants that might occur on the left hand side of the /∴ sign. Otherwise, we would be able to offer a proof of the obviously invalid argument:

11-8

Some cars built before 1970 were U.S.-manufactured Mustangs.
Therefore, John has a U.S.-manufactured Mustang.

1. (∃x)(Bx • Mx) /∴	Mj
2. Bj • Mj	(EI) (**incorrect!**!)
3. Mj • Bj	(2) Com.
4. Mj	(3) Simp.

One final note on these four new rules. Like the rules of inference in Chapter 9, all the rules of quantification must be applied to ***whole lines only***, otherwise we could prove the following argument to be valid.

11-9

If everything is a crow, then everything is black.
Some crows exist.
Therefore, everything is black.

1. (x)Cx ⊃ (x)Bx	
2. (∃x)Cx /∴ (x)Bx	
3. Ca	(2) EI
4. Ca ⊃ (x)Bx	(1) UI (**incorrect!!**)
5. Ca ⊃ B***y***	(4) UI (**incorrect!!**)
6. B***y***	(3)(5) MP[6]
7. (x)Bx	(6) UG

The Square of Opposition and Change of Quantifier Rules

We are now just about ready to try some formal proofs using our new additions to the 19 rules of propositional logic. We need to add one more rule, which is actually a series of variations of the equivalences noted in numbers 15 and 16 in the dictionary. Without this addition, we cannot prove valid arguments such as:

11-10

It is not true that all smokers develop lung cancer.
But all smokers are still health risks.
Therefore, some people who do not develop lung cancer are still health risks.

When we use the following translation:

1. ~(x)(Sx ⊃ Cx)
2. (x)(Sx ⊃ Hx) /∴ (∃x)(~Cx • Hx)

we need a rule to manipulate the first premise, and infer its simpler rendition, "Some smokers do not develop lung cancer." Before formalizing this rule, it will be helpful to review what has been

[6] Line 5 is unnecessary and is for illustration purposes only. We could conclude (x)Bx by MP from lines 3 & 4, and then correctly instantiate to B***y***. But the argument chain would still be broken by the invalid line 4.

traditionally called the ***Square of Opposition*** and compare it with its modern interpretation. Because it is so easy to confuse statements like "All cars built after 1970 had safety belts equipped at the factory," and "No cars built before 1970 had safety belts equipped at the factory," this pictorial device was constructed by logicians to keep track of the differences and the relationships between the basic *all*, *no*, *some*, and *some-not* categorical propositions. Consider the difference between these propositions:

A All smokers develop lung cancer.

E No smokers develop lung cancer.

I Some smokers develop lung cancer.

O Some smokers do not develop lung cancer.

From this we see that **O** propositions ***contradict*** **A** propositions. In other words, these propositions cannot both be true. If it were true that all smokers develop lung cancer, $(x)(Sx \supset Cx)$, then it would have to be false that some smokers do not develop lung cancer, $\sim(\exists x)(Sx \bullet \sim Cx)$; and if it were true that some smokers do not develop lung cancer, $(\exists x)(Sx \bullet \sim Cx)$, then it would have to be false that all smokers develop lung cancer, $\sim(x)(Sx \supset Cx)$. Similarly, **I** propositions ***contradict*** **E** propositions.

In the traditional Aristotelian interpretation of categorical propositions the common sense assumption was made that when we assert these propositions, we are making claims about actually existing things. Thus, there is a deductive relationship between **A** and **I** propositions, on the one hand, and **E** and **O** propositions on the other hand. If we are allowed to assume that at least one smoker exists, then if the **A** proposition is true, the **I** proposition must also be true; and, if the **E** proposition is true, then the **O** proposition must also be true. But notice that from a rigorous standpoint, we must make explicit the assumption that at least one smoker exists, because

All smokers develop lung cancer.
Therefore, some smokers develop lung cancer.

1. $(x)(Sx \supset Cx)$ $/\therefore$ $(\exists x)(Sx \bullet Cx)$

and

No smokers develop lung cancer
Therefore, some smokers do not develop lung cancer.

1. $(x)(Sx \supset \sim Cx)$ $/\therefore$ $(\exists x)(Sx \bullet \sim Cx)$

are both technically invalid! After instantiating, there is no way to derive validly a conjunction from a conditional statement.

1. $(x)(Sx \supset Cx)$ /∴ $(\exists x)(Sx \bullet Cx)$
2. $Sa \supset Ca$ (1) UI
3. $Sa \bullet Ca$????
4. $(\exists x)(Sx \bullet Cx)$ (3) EG

1. $(x)(Sx \supset {\sim}Cx)$ /∴ $(\exists x)(Sx \bullet {\sim}Cx)$
2. $Sa \supset {\sim}Ca$ (1) UI
3. $Sa \bullet {\sim}Ca$????
4. $(\exists x)(Sx \bullet {\sim}Cx)$ (3) EG

But with the assumption that there is at least one smoker, $(\exists x)Sx$, we can validly derive **I** propositions from **A** propositions, and **O** propositions from **E** propositions.

1. $(x)(Sx \supset Cx)$
2. $(\exists x)Sx$ /∴ $(\exists x)(Sx \bullet Cx)$
3. Sa (2) EI
4. $Sa \supset Ca$ (1) UI
5. Ca (3)(4) MP
6. $Sa \bullet Ca$ (3)(5) Conj.
4. $(\exists x)(Sx \bullet Cx)$ (6) EG

1. $(x)(Sx \supset {\sim}Cx)$
2. $(\exists x)Sx$ /∴ $(\exists x)(Sx \bullet {\sim}Cx)$
3. Sa (2) EI
4. $Sa \supset {\sim}Ca$ (1) UI
5. ${\sim}Ca$ (3)(4) MP
6. $Sa \bullet {\sim}Ca$ (3)(5) Conj.
4. $(\exists x)(Sx \bullet {\sim}Cx)$ (6) EG

In the traditional square of opposition the relationship between **A** and **I** propositions on the one hand, and between **E** and **O** propositions on the other hand, was said to be one of ***subalternation***. The **A** and **E** propositions were said to be ***superalterns*** and the **I** and **O** ***subalterns***.

Next, **A** and **E** propositions were said to be ***contraries***. A contrary relationship between propositions means that ***both cannot be true, but*** (unlike contradictions) ***both could be false***. It cannot be true both that all smokers develop lung cancer and that no smokers develop lung cancer. Thus, if it was true that all smokers develop lung cancer, then we would know that it is false that no smokers develop lung cancer, and if it was true that no smokers develop lung cancer, then we would know that it is false that all smokers develop lung cancer. Note that again we must assume $(\exists x)Sx$.

1. $(x)(Sx \supset Cx)$
2. $(\exists x)Sx$ $\quad\quad\quad$ $/\therefore\ \sim(x)(Sx \supset \sim Cx)$

1. $(x)(Sx \supset \sim Cx)$
2. $(\exists x)Sx$ $\quad$ $/\therefore\ \sim(x)(Sx \supset Cx)$

We will leave the proofs of these inferences for exercises. Note, however, that a contrary relationship is not as strong as a contradictory relationship. Because both **A** and **E** propositions could be false, if "All smokers develop lung cancer" is false, then we are uncertain about whether "No smokers develop lung cancer" is true; it could be true or false.

In the traditional square of opposition **I** and **O** propositions were said to be ***subcontraries***. A subcontrary relationship between propositions means that ***both cannot be false, but both could be true***. In other words, at least one is true. So, if it is false that some smokers develop lung cancer, then it is true that some smokers do not develop lung cancer. On the other hand, if it is false that some smokers do not develop lung cancer, then it is true that some do. Again, we must assume $(\exists x)Sx$ to prove this.

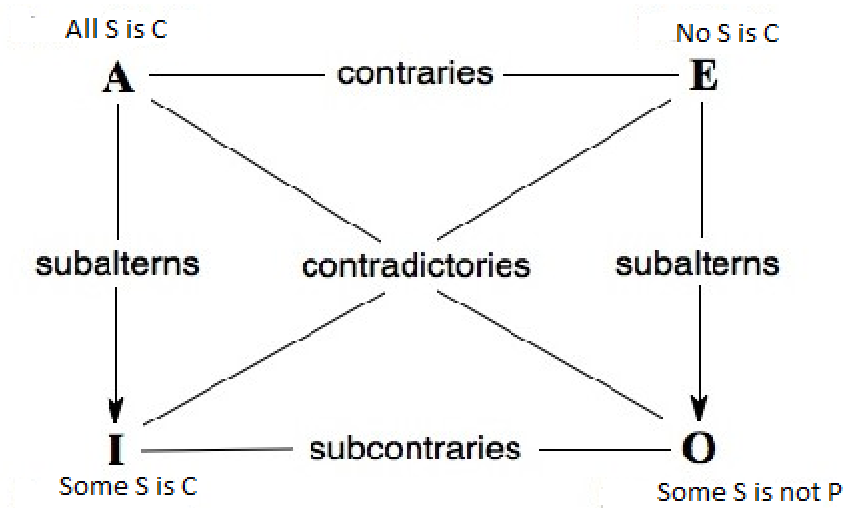
1. $\sim(\exists x)(Sx \bullet Cx)$
2. $(\exists x)Sx$ $\quad\quad\quad$ $/\therefore\ (\exists x)(Sx \bullet \sim Cx)$

1. $\sim(\exists x)(Sx \bullet \sim Cx)$
2. $(\exists x)Sx$ $\quad\quad\quad$ $/\therefore\ (\exists x)(Sx \bullet Cx)$

We will also leave the proofs of these inferences for exercises. Note that both propositions could be true. It could be true that some smokers develop lung cancer and some do not. (In fact we believe this is true based on the reasoning discussed in Chapter 3.) So in a subcontrary relationship, if an **I** or **O** proposition is known to be true, we cannot infer with certainty that its subcontrary is true or false. If it is true that "Some people on the football team have short hair," we cannot know deductively whether "Some people on the football team do not have short hair" is true. Nor can we know deductively whether it is false. We would have to discover the truth or falsity of this **O** proposition empirically.

We can now picture the ***Traditional Square of Opposition*** as follows:

11-11



Subalterns	=	**I** propositions can be deduced from **A** propositions; **O** propositions can be deduced from **E** propositions.
Contradictions	=	opposite truth value. If **A** is true, then **O** must be false; if **O** is true, then **A** must be false. If **E** is true, then **I** must be false; if **I** is true, then **E** must be false.
Contraries	=	at least one is false, both cannot be true, but both could be false. If **A** is true, then **E** must be false; if **E** is true, then **A** must be false. Both **A** and **E** could be false.
Subcontraries	=	at least one is true, not both are false. If **I** is false, then **O** must be true; if **O** is false, then **I** must be true. But both **I** and **O** could be true.

The utility of the traditional square of opposition, as in most pictures, is that it gives us a powerful tool for summarizing many inferences. For instance, if we knew that the **A** proposition, "All smokers are health risks," is true, then we immediately know that

1. The **O** proposition, "Some smokers are not health risks," is false via the relationship of contradiction with **A**. (If **A** is true, then **O** must be false.)

2. The **I** proposition, "Some smokers are health risks," is true, because it is a subaltern of **A** and a subcontrary of **O**. (If **A** is true, then **I** is true; if **O** is false, **I** is true.)

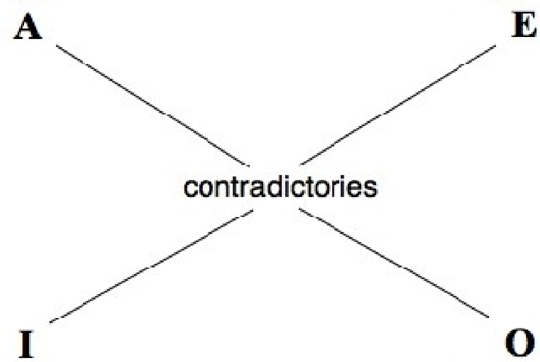
3. The **E** proposition, "No smokers are health risks," is false because of the relationship of contradiction with **I**, and the contrary relationship with **A**. (If **I** is true, then **E** is false; if **A** is true, then **E** is false.)

As neatly as the traditional square of opposition locks in these relationships, from a modern perspective there is a fatal flaw. We obviously cannot assume in all uses of categorical propositions that the things referred to by the terms actually exist. In fact, a moment's reflection on the contents of tabloid newspaper articles while standing in your local supermarket check-out line will demonstrate that questionable assertions about existence are made all the time, such as articles on a woman having a Martian baby, some of Hitler's men returning to Earth from an astronautical adventure that began in 1944, and a claim to be able to explain the sudden reduction of the number of frogs throughout the world by an arrival of extraterrestrial creatures who were eating the frogs. That we can intelligibly, but not I believe intelligently, make statements such as, "Some Martian babies exist," and "All extraterrestrial creatures eat frogs," does not mean that such things actually exist.[7]

Thus, modern logicians claim that the Aristotelian traditional square of opposition commits an ***existential fallacy*** when it is applied to propositions that make assertions about things that do not actually exist. In the above discussion, notice what would be left in terms of valid deductive relations to the square of opposition if we cannot make the assumption that some smokers exist, $(\exists x)Sx$. This assumption was needed to show the validity of the subalternation, contrary, and subcontrary relationships. Thus, the only relationship that remains in what is called the ***Modern Square of Opposition*** is the relationship of contradiction between **A** and **O** propositions and **E** and **I** propositions.

[7] Incidentally, a much better possible scientific explanation for the disappearance of frogs is that frogs are amphibians, live partly in water, and hence have water permeable skins. Reproducing in water, their eggs are thus hypersensitive to environmental pollution and increased ultraviolet radiation due to ozone depletion. This explanation is corroborated by laboratory experiments and the fact that other species of amphibians are also disappearing at an alarming rate.

11-12



This result is not trivial. It reflects not only a major cultural difference between our age and the cozy, secure universe that Aristotle and later people in the Middle Ages accepted—we have a much less certain belief environment—but this result also underscores again the importance of valid deductive analysis for testing our beliefs. What cannot be known with deductive certainty must be inductively tested. Recall from our discussion in Chapter 2 that previous philosophers in Western culture thought that they could deductively prove the foundational beliefs of their societies. Recall also from our previous discussion in this chapter that during the Middle Ages philosophers and theologians thought they could prove the existence of their version of God from words alone, that is, that from the mere contemplation of their definition of God, they could deduce His existence.

Advances in logic and science, and the general awareness of the practices and beliefs in other cultures, have epistemologically pruned our world of such certainties. To make this statement is not to imply necessarily that one cannot be rational and believe in some of the same things people did in the Middle Ages, such as the existence of God. Rather, it only implies that one must work harder. From a modern perspective, there are no logical proofs of the existence of God.[8] However, as has been repeatedly emphasized in this book, this ought not to leave us with a belief-emptiness, with relativism and nihilism. Simply put, our situation is more challenging, but it is not hopeless. What logical analysis shows cannot be certain is left for the bootstrap process of inductive reasoning and the on-going process of trying to make our inductions stronger.

---

[8] According to Kant, this realization allows for the possibility of a rational faith.

The bottom line of this analysis is that we can now see clearly what must be added to our quantification rules to be able to deal with arguments such as **11-10**. First let's summarize symbolically the contradictory relationships.

$(x)(Sx \supset Cx) \equiv \sim(\exists x)(Sx \bullet \sim Cx)$	The truth of an **A** proposition is logically equivalent to the negation of an **O** proposition.
$\sim(x)(Sx \supset Cx) \equiv (\exists x)(Sx \bullet \sim Cx)$	The negation of an **A** proposition is logically equivalent to the truth of an **O** proposition.
$(\exists x)(Sx \bullet Cx) \equiv \sim(x)(Sx \supset \sim Cx)$	The truth of an **I** proposition is logically equivalent to the negation of an **E** proposition.
$\sim(\exists x)(Sx \bullet Cx) \equiv (x)(Sx \supset \sim Cx)$	The negation of an **I** proposition is logically equivalent to the truth of an **E** proposition.

From these equivalences via a little work with some of the rules of replacement we can derive the following equivalences: (We will save the work for exercises)

$(x)(Sx \supset Cx) \equiv \sim(\exists x)\sim(Sx \supset Cx)$

$\sim(x)(Sx \supset Cx) \equiv (\exists x)\sim(Sx \supset Cx)$

$(\exists x)(Sx \bullet Cx) \equiv \sim(x)\sim(Sx \bullet Cx)$

$\sim(\exists x)(Sx \bullet Cx) \equiv (x)\sim(Sx \bullet Cx)$

Finally, we can summarize these equivalences, using our variable Ψ, as one set of ***Change of Quantifier Rules*** (**CQ**). Recall that Ψ stands for any terms whatsoever.

$(x)(\Psi x) \equiv \sim(\exists x)\sim(\Psi x)$ $\qquad$ $\sim(x)(\Psi x) \equiv (\exists x)\sim(\Psi x)$

$(\exists x)(\Psi x) \equiv \sim(x)\sim(\Psi x)$ $\qquad$ $\sim(\exists x)(\Psi x) \equiv (x)\sim(\Psi x)$

Essentially, these rules formalize a set of relationships part of which we noted above when discussing numbers 15 and 16 in the dictionary. Completing this analysis, we can now see that saying,

Everything is perfect, $(x)Px$ $\equiv$ It is not true that there is something that is not perfect, $\sim(\exists x)\sim Px$

It is not true that everything is perfect, $\sim(x)Px$ $\equiv$ Some things are not perfect, $(\exists x)\sim Px$

Some things are perfect,
(∃x)Px ≡ It is not true that nothing is perfect, ~(x)~Px

It is not true that some things are perfect, ~(∃x)Px ≡ Nothing is perfect, (x)~Px

In using the change of quantifier rules mechanically, all you need to remember is this: If you see (x), you can change to ~(∃x)~ and vice versa; if you see (∃x), you can change to ~(x)~ and vice versa; if you see ~(x) you can change to (∃x)~ and vice versa; if you see ~(∃x) you can change to (x)~ and vice versa.

We can now offer a proof for **11-10** as follows:

1. ~(x)(Sx ⊃ Cx)	
2. (x)(Sx ⊃ Hx) /∴ (∃x)(~Cx • Hx)	
3. (∃x)~(Sx ⊃ Cx)	(1) CQ
4. (∃x)~(~~Sx ⊃ Cx)	(3) DN
5. (∃x)~(~Sx v Cx)	(4) Impl.
6. (∃x)(~~Sx • ~Cx)	(5) De M.
7. (∃x)(Sx • ~Cx)	(6) DN
8. Sa • ~Ca	(7) EI
9. Sa ⊃ Ha	(2) UI
10. Sa	(8) Simp.
11. Ha	(9)(10) MP
12. ~Ca	(8) Com. + Simp.
13. ~Ca • Ha	(12)(11) Conj.
14. (∃x)(~Cx • Hx)	(13) EG

Before doing the following exercises summarize all the quantification rules, including the change of quantifier rules, on one sheet of paper. Be sure to include notes on the restrictions.

Exercises II

Justify each correct step in the following. Put an X for any line that involves the incorrect application of a rule.

#1

1. (x)(Ax ⊃ Bx)
2. ~(∃x)(Bx • Cx) /∴ (x)(Ax ⊃ ~Cx)
3. (x)~(Bx • Cx)
4. (x)(~Bx v ~Cx)
5. (x)(~~Bx ⊃ ~Cx)
6. (x)(Bx ⊃ ~Cx)
7. Ay ⊃ By
8. By ⊃ ~Cy
9. Ay ⊃ ~Cy
10. (x)(Ax ⊃ ~Cx)

#2

1. (x)(Bx ⊃ Cx)
2. (∃x)(Bx • ~Hx) /∴ (∃x)(Cx • ~Hx)
3. Bd • ~Hd
4. Bd
5. Bd ⊃ Cd
6. Cd
7. ~Hd • Bd
8. ~Hd
9. Cd • ~Hd
10. (∃x)(Cx • ~Hx)

#3*

1. ~(x)(Fx ⊃ ~Ax)
2. (∃x)(Fx • Ox) /∴ (∃x)(Ax • Ox)
3. (∃x)~(Fx ⊃ ~Ax)
4. (∃x)~(~~Fx ⊃ ~Ax)
5. (∃x)~(~Fx v ~Ax)
6. (∃x)(~~Fx • ~~Ax)
7. (∃x)(Fx • ~~Ax)
8. (∃x)(Fx • Ax)
9. Fa • Aa
10. Fa • Oa
11. Aa • Fa
12. Oa • Fa
13. Aa
14. Oa
15. Aa • Oa
16. (∃x)(Ax • Ox)

#4

1. (x)[(Dx • Cx) ⊃ Ax]
2. (∃x)(Cx • ~Ax) /∴ ~(x)Dx
3. Cm • ~Am
4. (Dm • Cm) ⊃ Am
5. ~Am • Cm
6. ~Am
7. ~(Dm • Cm)
8. ~Dm v ~Cm
9. ~Cm v ~Dm
10. Cm
11. ~~Cm
12. ~Dm
13. (∃x)~Dx
14. ~(x)Dx

#5

1. (x)(Bx v Cx)
2. (x)(Bx ⊃ Cx) /∴ (∃x)Cx
3. By v Cy
4. ~By ⊃ Cy
5. By ⊃ Cy
6. ~Cy ⊃ ~By

#6

1. (∃x)~(Bx • Cx) /∴ (x)(Bx ⊃ ~Cx)
2. (∃x)(~Bx v ~Cx)
3. (∃x)(~~Bx ⊃ ~Cx)
4. (∃x)(Bx ⊃ ~Cx)
5. Ba ⊃ ~Ca
6. (x)(Bx ⊃ ~Cx)

7. ~Cy ⊃ Cy
8. Cy v Cy
9. Cy
10. (x)Cx

#7*

1. (∃x)(Ax • Fx) /∴ ~(∃x)(Ax ⊃ ~Fx)
2. Ay • Fy
3. ~~(Ay • Fy)
4. ~(~Ay v ~Fy)
5. ~(~~Ay ⊃ ~Fy)
6. ~(Ay ⊃ ~Fy)
7. (x)~(Ax ⊃ ~Fx)
8. ~(∃x)(Ax ⊃ ~Fx)

#8

1. (∃x)~(Gx • Hx)
2. ~(x)(Gx • Hx) ⊃ (∃x)(Tx • ~Px)
3. (x)(Cx ⊃ Px) /∴ (∃x)(Tx • ~Cx)
4. (∃x)~(Gx • Hx) ⊃ (∃x)(Tx • ~Px)
5. (∃x)(Tx • ~Px)
6. Ta • ~Pa
7. Ca ⊃ Pa
8. ~Pa • Ta
9. ~Pa
10. ~Ca
11. Ta
12. Ta • ~Ca
13. (∃x)(Tx • ~Cx)

#9

1. ~(∃x)(Kx • ~Yx)
2. (x)[(Kx • Yx) ⊃ Zx] /∴ (x)(Kx ⊃ Zx)
3. (x)~(Kx • ~Yx)
4. (x)(~Kx v ~~Yx)
5. (x)(~Kx v Yx)
6. (x)(~~Kx ⊃ Yx)
7. (x)(Kx ⊃ Yx)
8. Ky ⊃ Yy
9. (Ky • Yy) ⊃ Zy
10. (Yy • Ky) ⊃ Zy
11. Yy ⊃ (Ky ⊃ Zy)
12. Ky ⊃ (Ky ⊃ Zy)
13. (Ky • Ky) ⊃ Zy
14. Ky ⊃ Zy
15. (x)(Kx ⊃ Zx)

#10

1. (x)[(Ax v Cx) ⊃ Dx]
2. ~(∃x)(~Ax • ~Cx)
3. (∃x)(~Dx v Px) /∴ (∃x)Px
4. ~Da v Pa
5. (x)~(~Ax • ~Cx)
6. (x)(~~Ax v ~~Cx)
7. (x)(Ax v ~~Cx)
8. (x)(Ax v Cx)
9. (x)(~Ax ⊃ Cx)
10. ~Aa ⊃ Ca
11. (Aa v Ca) ⊃ Da
12. (~Aa ⊃ Ca) ⊃ Da
13. Da
14. ~~Da
15. Pa
16. (∃x)Px

Exercise III

Write an essay, complete with a symbolic demonstration similar to 11-7 or 11-9, explaining why it is invalid to conclude that "Some students are faculty" from the premise "Not only students but faculty as well attended the initiation dance." (#17, Ex. I)

Exercises IV

Prove the contrary and subcontrary relationships in the Traditional Square of Opposition that were left unproved.

Contraries: If an **A** proposition is true, then an **E** proposition is false. If an **E** proposition is true, then an **A** proposition is false. Prove:

1. $(x)(Sx \supset Cx)$
2. $(\exists x)Sx$ $\quad /\therefore \sim(x)(Sx \supset \sim Cx)$

1. $(x)(Sx \supset \sim Cx)$
2. $(\exists x)Sx$ $\quad /\therefore \sim(x)(Sx \supset Cx)$

Subcontraries: If an **I** proposition is false, then an **O** proposition is true. If an **O** proposition is false, then an **I** proposition is true. Prove:

1. $\sim(\exists x)(Sx \bullet Cx)$
2. $(\exists x)Sx$ $\quad /\therefore (\exists x)(Sx \bullet \sim Cx)$

1. $\sim(\exists x)(Sx \bullet \sim Cx)$
2. $(\exists x)Sx$ $\quad /\therefore (\exists x)(Sx \bullet Cx)$

Exercises V

Prove the equivalences discussed above by proving the following.

#1* $\quad (x)(Sx \supset Cx) \equiv \sim(\exists x)\sim(Sx \supset Cx)$, because

$\sim(\exists x)\sim(Sx \supset Cx) \equiv \sim(\exists x)(Sx \bullet \sim Cx)$, so prove:

1. $\sim(\exists x)\sim(Sx \supset Cx)$ / $\therefore$ $\sim(\exists x)(Sx \bullet \sim Cx)$

and

1. $\sim(\exists x)(Sx \bullet \sim Cx)$ / $\therefore$ $\sim(\exists x)\sim(Sx \supset Cx)$

#2 $\sim(x)(Sx \supset Cx) \equiv (\exists x)\sim(Sx \supset Cx)$, because

$(\exists x)\sim(Sx \supset Cx) \equiv (\exists x)(Sx \bullet \sim Cx)$, so prove:

1. $(\exists x)\sim(Sx \supset Cx)$ / $\therefore$ $(\exists x)(Sx \bullet \sim Cx)$

and

1. $(\exists x)(Sx \bullet \sim Cx)$ / $\therefore$ $(\exists x)\sim(Sx \supset Cx)$

#3 $(\exists x)(Sx \bullet Cx) \equiv \sim(x)\sim(Sx \bullet Cx)$, because

$\sim(x)\sim(Sx \bullet Cx) \equiv \sim(x)(Sx \supset \sim Cx)$, so prove:

1. $\sim(x)\sim(Sx \bullet Cx)$ / $\therefore$ $\sim(x)(Sx \supset \sim Cx)$

and

1. $\sim(x)(Sx \supset \sim Cx)$ / $\therefore$ $\sim(x)\sim(Sx \bullet Cx)$

#4 $\sim(\exists x)(Sx \bullet Cx) \equiv (x)\sim(Sx \bullet Cx)$, because

$(x)\sim(Sx \bullet Cx) \equiv (x)(Sx \supset \sim Cx)$, so prove:

1. $(x)\sim(Sx \bullet Cx)$ / $\therefore$ $(x)(Sx \supset \sim Cx)$

and

1. $(x)(Sx \supset \sim Cx)$ / $\therefore$ $(x)\sim(Sx \bullet Cx)$

Exercises VI

Translate each of the following. Then when you are sure you have the correct translation, use the change of quantifier relationships and the rules of replacement to streamline each translation.

Eg.

It is not true that there are some smokers who are not health risks. (Sx = x is a smoker; Hx = x is a health risk.)

~(∃x)(Sx • ~Hx)

1. ~(∃x)(Sx • ~Hx)	
2. (x)~(Sx • ~Hx)	(1)CQ
3. (x)(~Sx v ~~Hx)	(2) De M.
4. (x)(~Sx v Hx)	(3) DN
5. (x)(~~Sx ⊃ Hx)	(4) Impl.
6. (x)(Sx ⊃ Hx)	(5) DN

Result as shown by line 6: All smokers are health risks.

1. It is not true that all marijuana users abuse other drugs. (Mx = x is a marijuana user; Ax = x abuses other drugs.)

2.* In the U.S. it is not true that some judges either do not go to law school or do not pass the bar exam. (Jx = x is a judge in the U.S.; Lx = x goes to law school; Bx = x passes the bar exam.)

3. Not all smokers die of lung cancer, but it is not true that some are not health risks. (Sx = x is a smoker; Dx = x dies of lung cancer; Hx = x is a health risk.)

4. It is not true that all students who passed the final did not pass the course. (Fx = students who passed the final; Cx = students who passed the course.)

5. Although it is not true that only politicians are rich, there does not exist a single politician who is not rich. (Rx = x is rich; Px = x is a politician)

Exercises VII

Translate each of the following arguments. When you are sure the translation is correct, provide a formal proof for each.

#1. Everyone on the football team must have short hair. John does not have short hair. So, John is not on the football team. (Fx = x is on the football team; Sx = x has short hair; j = John.)

#2. Only physicians can write prescriptions. Anyone who does not write prescriptions does not give business to pharmacists. Thus, only physicians give business to pharmacists. (Px = x is a physician; Wx = x writes prescriptions; Bx = x gives business to pharmacists.)

#3.* Professors are either enthusiastic or they do not motivate students. Not all professors fail to motivate students. So, some professors are enthusiastic. (Px = x is a professor; Ex = x is enthusiastic; Mx = x motivates students.)

#4. Those who ignore history are condemned to repeat it. Those who act for short term gain ignore history. Some politicians act for short term gain. Therefore, some politicians are condemned to repeat history. (Ix = ignores history; Cx = x is condemned to repeat history; Px = x is a politician; Sx = x acts for short term gain.)

#5. All students are eligible for tuition waivers if and only if they are not Freshman and have at least a B grade point average. Kriegle is a student with a B grade point average, but is not eligible. So, Kriegle must be a Freshman. (Sx = x is a student; Ex = x is eligible for a tuition waiver; Fx = x is a Freshman; Bx = x has at least a B grade point average; k = kriegle.)

#6. All supporters of abortion are either liberals or moderates. But no moderate supports abortion after the first trimester. Only abortion supporters support abortions after the first trimester. So, only liberals support abortions after the first trimester. (Ax = x supports abortion; Lx = x is a liberal; Mx = x is a moderate; Fx = x supports abortion after the first trimester.)

#7. Only persons over 62 receive Social Security retirement benefits. Any person over 62 is also eligible for a one time capital gains tax reduction. But some people are eligible for a one time capital gains tax reduction who are not over 62. Accordingly, although there is no person who receives social security who is not eligible for the one time capital gains tax reduction, it is not true that only people over 62 are eligible for the one time capital gains tax reduction. (Ox = x is over 62; Sx = x receives Social Security retirement benefits; Ex = x is eligible for a one time capital gains tax reduction.)

#8. It is not true that every citizen in the United States owns more than one gun. The population of the United States is about two hundred and fifty million. The number of guns in the United States is six hundred million. If the population of the United States is about two hundred and fifty million and the number of guns in the United States is six hundred million, then some citizens of the Unites States own more than one gun. If some citizens of the United States do not own more than one gun and some citizens of the United States do own more than one gun, then the gun laws are flawed and should be changed. Hence, it is not true that there is a gun law that is not flawed or should not be changed. (Ux = x is a citizen of the United States; Ox = x owns more than one gun; P = the population of the United States is about two hundred and fifty million; N = the number of guns in the United States is over six hundred million; Gx = x is a gun law; Fx = x is flawed; Cx = x should be changed.)

#9.* Not all voters make intelligent choices during elections. If some voters do not make intelligent choices during elections, then some dishonest politicians will take advantage of fallacious thinking. But some politicians are honest. If some politicians are honest and some politicians are not honest, then all voters should be critical thinkers in a democracy. It follows that it is not true that some voters should not be critical thinkers in a democracy. (Vx = x is a voter; Ix = x makes intelligent choices during elections; Hx = x is honest; Px = x is a politician; Ax = x takes advantage of fallacious thinking; Cx = x should be a critical thinker in a democracy.)

#10. No Democratic politician is both a liberal and an advocate of President Obama’s version of the health care bill. But all Democratic politicians are either liberals or advocates of President Obama’s version of the health care bill. So it follows that no Democratic politician is a liberal if they are an advocates of President Obama’s version of the health care bill, and an advocate of President Obama’s version of the health care bill if they are a liberal; whereas, all Democratic politicians who are not liberals are advocates of President Obama’s version of the health care bill and all Democratic politicians who are not advocates of President Obama’s version of the health care bill are liberals. (Dx = x is a Democratic politician; Lx = x is a liberal; Ax = x is an advocate of President Obama’s version of the health care bill.)

Does the conclusion, "All Democratic politicians are such that they are liberals if and only if they are not advocates of President Obama's version of the health care bill and advocates of President Obama's version of the health care bill if and only if they are not liberals" also follow from these premises?

Final Note

The initial quantification steps we have taken are the proper place to conclude a standard introduction to logic. However, as noted at the beginning of this chapter there are many more symbolic techniques that logicians have devised in their attempt to help us weave our way through the rich and often ambiguous experience of life. Since this book began with a quotation from Abraham Lincoln, let us conclude this chapter with just a taste of the next level of quantification by showing what a translation of this famous statement would look like.

Recall that, according to Lincoln, "You can fool some of the people all of the time, and all of the people some of the time, but you cannot fool all of the people all of the time." In order to translate this statement, we must be able to translate relationships, such as Kila is taller than Rubin, Tran gained a friend, John is fooled all the time, and so on. Here is how these examples could be translated:

Kila is taller than Rubin = Tkr = k is T(aller) than r.

Tran gained a friend = $(\exists x)(Fx \bullet Gtx)$ = There is an x such that x is a F and t G(ained) x.

John is fooled all the time = $(x)(Tx \supset Fjx)$ = Given any x, if x is a T, then j is F(ooled) at x.

Accordingly, here is how the Lincoln quotation would be rendered:

"There is an x such that x is a person and given any y if y is a time then x is fooled at y, and there is a y such that y is a time and given any x if x is a person then x is fooled at y, but there is a y and there is an x such that y is a time and x is a person, and x is not fooled at y."

Hence, with Ty = y is a time, Px = x is a person, and Fxy = x is fooled at y, we have

$$\{(\exists x)[Px\bullet(y)(Ty \supset Fxy)]\bullet(\exists y)[Ty\bullet(x)(Px \supset Fxy)]\}\bullet(\exists y)(\exists x)[(Ty\bullet Px)\bullet\sim Fxy)]$$

Life gets complicated. So does symbolic logic. But let us hope that the basic logical skills covered in this book, the general analytical discipline encouraged, and the perspective of obtaining reliable beliefs in an uncertain but precious world will help decrease the number of people fooled in the twenty-first century.

Answers to Starred Exercises

Exercises I

3. $(x)(Cx \supset Tx)$

8. $\sim(x)(Px \supset Mx)$

13. $(x)[(Wx \lor Hx) \supset Mx]$ Notice (v) because we don't want this to mean that something is both a whale and a human being!

17. $(\exists x)(Sx \bullet Ax) \bullet (\exists x)(Ax \bullet \sim Sx) \bullet (\exists x)(Fx \bullet Ax)$
Notice the assumption that some students attended.

Exercises II

#3

1. $\sim(x)(Fx \supset \sim Ax)$	
2. $(\exists x)(Fx \bullet Ox)$ /∴ $(\exists x)(Ax \bullet Ox)$	
3. $(\exists x)\sim(Fx \supset \sim Ax)$	(1) CQ
4. $(\exists x)\sim(\sim\sim Fx \supset \sim Ax)$	(3) DN
5. $(\exists x)\sim(\sim Fx \lor \sim Ax)$	(4) Impl.
6. $(\exists x)(\sim\sim Fx \bullet \sim\sim Ax)$	(5) De M.
7. $(\exists x)(Fx \bullet \sim\sim Ax)$	(6) DN
8. $(\exists x)(Fx \bullet Ax)$	(7) DN
9. $Fa \bullet Aa$	(8) EI
10. $Fa \bullet Oa$	(2) EI **X (EI restriction violated)**
11. $Aa \bullet Fa$	(9) Com
12. $Oa \bullet Fa$	(10) Com
13. Aa	(11) Simp.
14. Oa	(12) Simp.
15. $Aa \bullet Oa$	(13)(14) Conj.
16. $(\exists x)(Ax \bullet Ox)$	(15) EG

#7

1. $(\exists x)(Ax \bullet Fx)$ /∴ $\sim(\exists x)(Ax \supset \sim Fx)$	
2. $Ay \bullet Fy$	(1) EI **X (EI restriction violated)**

3. ~~(Ay • Fy) (2) DN
4. ~(~Ay v ~Fy) (3) De M.
5. ~(~~Ay ⊃ ~Fy) (4) Impl.
6. ~(Ay ⊃ ~Fy) (5) DN
7. (x)~(Ax ⊃ ~Fx) (6) UG
8. ~(∃x)(Ax ⊃ ~Fx) (7) CQ

Exercises V

#1

1. ~(∃x)~(Sx ⊃ Cx) /∴ ~(∃x)(Sx • ~Cx)
2. ~(∃x)~(~~Sx ⊃ Cx)(1) DN
3. ~(∃x)~(~Sx v Cx) (2) Impl.
4. ~(∃x)(~~Sx • ~Cx) (3) De M.
5. ~(∃x)(Sx • ~Cx) (4) DN

1. ~(∃x)(Sx • ~Cx) /∴ ~(∃x)~(Sx ⊃ Cx)
2. ~(∃x)(~~Sx • ~Cx) (1) DN
3. ~(∃x)~(~Sx v Cx) (2) De M.
4. ~(∃x)~(~~Sx ⊃ Cx)(3) Impl.
5. ~(∃x)~(Sx ⊃ Cx) (4) DN

Exercises VI

#2. ~(∃x)[Jx • (~Lx v ~Bx)]

1. ~(∃x)[Jx • (~Lx v ~Bx)]
2. (x)~[Jx • (~Lx v ~Bx)] (1) CQ
3. (x)[~Jx v ~(~Lx v ~Bx)] (2) De M.
4. (x)[~~Jx ⊃ ~(~Lx v ~Bx)] (3) Impl.
5. (x)[Jx ⊃ ~(~Lx v ~Bx)] (4) DN
6. (x)[Jx ⊃ (~~Lx • ~~Bx)] (5) De M.
7. (x)[Jx ⊃ (Lx • ~~Bx)] (6) DN
8. (x)[Jx ⊃ (Lx • Bx)] (7) DN

Result as shown by line 8: All judges in the U. S. must go to law school and pass the bar exam.

Exercises VII

#3.

1. (x)[Px ⊃ (Ex v ~Mx)]	
2. ~(x)(Px ⊃ ~Mx) /∴ (∃x)(Px • Ex)	
3. (∃x)~(Px ⊃ ~Mx)	(2) CQ
4. (∃x)~(~~Mx ⊃ ~Px)	(3) Trans.
5. (∃x)~(~Mx v ~Px)	(4) Impl.
6. (∃x)(~~Mx • ~~Px)	(5) De M.
7. ~~Ma • ~~Pa	(6) EI
8. [Pa ⊃ (Ea v ~Ma)]	(1) UI
9. ~~Pa • ~~Ma	(7) Com.
10. ~~Pa	(9) Simp.
11. Pa	(10) DN
12. Ea v ~Ma	(8)(11) MP
13. ~Ma v Ea	(12) Com.
14. ~~Ma	(7) Simp.
15. Ea	(13)(14) DS
16. Pa • Ea	(11)(15) Conj.
17. (∃x)(Px • Ex)	(16) EG

#9.

1. ~(x)(Vx ⊃ Ix)	
2. (∃x)(Vx • ~Ix) ⊃ (∃x)[(~Hx • Px) • Ax]	
3. (∃x)(Px • Hx)	
4. [(∃x)(Px • Hx) • (∃x)(Px • ~Hx)] ⊃ (x)(Vx ⊃ Cx) /∴ ~(∃x)(Vx • ~Cx)	
5. (∃x)~(Vx ⊃ Ix)	(1) CQ
6. (∃x)~(~~Vx ⊃ Ix)	(5) DN
7. (∃x)~(~Vx v Ix)	(6) Impl.
8. (∃x)(~~Vx • ~Ix)	(7) De M.
9. (∃x)(Vx • ~Ix)	(8) DN
10. (∃x)[(~Hx • Px) • Ax]	(2)(9) MP
11. (~Ha • Pa) • Aa	(10) EI
12. ~Ha • Pa	(11) Simp.
13. Pa • ~Ha	(12) Com.
14. (∃x)(Px • ~Hx)	(13) EG
15. (∃x)(Px • Hx) • (∃x)(Px • ~Hx)	(3)(14) Conj.
16. (x)(Vx ⊃ Cx)	(15)(4) MP

17. $(x)(\sim\sim Vx \supset Cx)$	(16) DN
18. $(x)(\sim Vx \vee Cx)$	(17) Impl.
19. $(x)(\sim Vx \vee \sim\sim Cx)$	(18) DN
20. $(x)\sim(Vx \bullet \sim Cx)$	(19) De M.
21. $\sim(\exists x)(Vx \bullet \sim Cx)$	(20) CQ