Chapter 8: Bit-brains, Logical Connectives, and Truth Tables

Introduction

In the previous chapter we learned symbolic translations for the five key logical connectives *not* (~), *and* (•), or (v), *if . . . then* (⊃), and *if and only if* (≡). In this chapter we will learn our first symbolic method for judging arguments to be valid or invalid. To accomplish this, we must be able to devise a way of capturing symbolically the essential meaning of these words and phrases. As we have seen, logicians have discovered that a key element in deciding whether an argument is valid or invalid is the argument's structure or form.

For instance, in Chapter 1 we learned that arguments with these patterns (using the symbols we learned from Chapter 7) are always valid:

A ⊃ B
A /∴ B

A ⊃ B
~B /∴ ~A

And arguments with these patterns are always invalid:

A ⊃ B
B /∴ A

A ⊃ B
~A /∴ ~B

In this chapter we will see that it is the particular configuration of statements, component statements, and logical connectives that determine an argument's structure. In the above arguments the patterns of relationships between the first and second premises determine validity or invalidity. When the consequent of an (⊃) statement is negated in another premise (second example), concluding the negation of the antecedent will always be valid. When the antecedent is negated (fourth example) in another premise, concluding the negation of the consequent will always be invalid.

Our goal will be to work from the smallest commonsense parts of the way we use our language to analyze more difficult reasoning trails. Thus, we now need to provide rules of usage for these key logical connectives and some method of representing or picturing these rules in our symbolic language. This task is more technologically and philosophically interesting than it may first seem.

By the time we are adults we take much for granted. So much so that there is not near the amount of mystery, apparent magic, excitement, and sense of continual discovery that existed for us when we were children. For instance, we take for granted that we know what the words *table*, *pen*, *eraser*, and *father* refer to. But consider the possible initial difficulty you had with these words when you were a child. Consider the situation of a mother teaching her child what the word *pen* means. Suppose she holds a pen in her hands and then, pointing to it, says "pen" several times. Initially, how does the child know what the mother is referring to? Viewed from the fresh and imaginative perspective of a child, virtually an infinite number of things could be identified by the child as the denotative meaning (Chapter 2) of the strange new word *pen*. The child might notice that the mother is holding the pen at a particular angle, that it is being held against a particular background, that the mother is holding her hand in a certain way, that she is gesturing in a strange way; that this new object is being held this way in a particular room, at a particular time, on a particular day; that when she makes this gesture no one else is in the room, or someone else is in the room, or the family dog is in the room, or that there are or are not chairs in the room, or that her mother has a particular expression on her face. From the open perspective of a child not yet locked into a "correct" view of things, any of these circumstances could mean *pen*. Furthermore, because the mother may utter the word *pen* several times when gesturing emphatically with the pen in her hand, the child may also think that any one of these circumstances may be the referent for the repetitive phrase "pen . . . pen"! Eventually, through surprisingly little trial and error and use of the word *pen* in different contexts, we understand what is actually a very abstract word.

Consider, too, the greater difficulty of learning words such as *and* and *or*. At least pens are physical objects. With words such as *and* and *or* parents have nothing to point to when trying to teach a child their different meanings. There are no objects or little creatures running around in the world called *and* and *or* that we could trip over. Eventually we understand the difference between saying, "Lisa, go outside to see if the cat *and* the dog are there waiting for dinner," and "Lisa, go outside to see if the cat *or* the dog is there waiting for dinner." What do we eventually understand when we learn to use these logical connectives correctly?

Symbolic Pictures of Logical Connectives: And (•)*, Or* (v)*, and Not* (~)

Let's begin with not (**~**) and statements we call **negations** . If someone were to say "Alice is going to the party" (A), and I later found this statement to be true (T), then I would know the statement "Alice is not going to the party" (~A) to be false

(F). On the other hand, if I found out that the statement "Alice is not going to the party" is true, then I would know the statement "Alice is going to the party " to be false . This is how we use ‘not,’ and this will be our rule for ‘not’: If a statement (A) is true, then the negation of that statement (~A) is false, and if a statement (~A) is true, then (A) is false. This is how we picture this rule symbolically.

A	~A
T	F
F	T

When we use the word *not* correctly this is what we mean: **the negation of a true statement is false (~T = F), and the negation of a false statement is true (~F = T)**. Because this rule of usage applies to any statement using a negation—statements about AIDS, presidential elections, the environment, or the passing of a final exam, not just a statement about Alice going to a party—logicians introduce the technique of using a **variable** to summarize the rule. The variables used in logic (p, q, r), play a similar role to the variables (x, y, z) used in algebra. In mathematics it would take a long time to write every number (impossible in fact) to explain that in adding two numbers, it does not matter which number is first and which one is second. So we summarize this rule with the use of variables: (x + y) = (y + x). Similarly it would be a waste of our valuable time to endlessly write the above little table for every possible statement. Instead we summarize the rule with a variable. Here is the rule for negation where the small **p** stands for **any statement whatsoever**:

Table (rule) for Negation (~)

p	~p
T	F
F	T

Next, we have the connective and (•). Statements using this connective you will recall are called **conjunctions**. If someone said "Alice and Barbara are going to the party" (A • B) and we found that it is true that Alice is going and true that Barbara is going, then we would know this statement to be true. But if we found that although Alice is going, Barbara is not, then we would know the above statement to be false, because the statement claims both are going, and in this case only Alice is going. We would also know the above statement to be false in the case Alice is not going, even though Barbara is, and in the case in which neither Alice nor Barbara are going. This, then, will be the rule for *and*: **A conjunction (A • B) is true when all the components are true, and false otherwise**. The following is how we picture this rule symbolically:

A	**B**	**A•B**
T	T	T
T	F	F
F	T	F
F	F	F

Furthermore, because this table result would be the same for the statement "Nguyen passed the final exam, and he also passed the course" (F • C), we picture the conjunction rule for any statement whatsoever using variables as follows:

Table (rule) for conjunction (•)

p	q	p•q
T	T	T
T	F	F
F	T	F
F	F	F

To conclude this section, we have the logical connective or (v) and statements called **disjunctions**. As you recall from the previous chapter, providing a symbolic picture of this connective is complicated by the fact that we find two different usages of this connective in our language. First, there is the **inclusive** sense of *or*, such as when someone says "Alice or Barbara is going to the party," and they mean "Well, I'm not sure. They might both be going, but I think at least either Alice is going or Barbara is going." This sense of *or* means **one or the other, possibly both**. Secondly, there is an **exclusive** sense of *or*, such as the case where a menu at a restaurant states that you may have soup or salad and the intention is clear that you may have **one or the other but not both**.

We will choose to picture only the inclusive sense with the (v) symbol for two reasons. First of all, any statement that has the exclusive sense intended can be pictured using the inclusive symbol plus other symbols such as *not* (~) and *and* (•). For instance, if a menu stated that each customer may have pea soup or salad we could picture this in the following way: (P v S) • ~(P • S). That is, pea soup or salad, but not both pea soup and salad. Second, there is no reason to complicate our translation process by adding another symbol. So, for the sake of convenience we will use the symbol (v) to stand for the inclusive sense of *or*.

So, if someone said (intending the inclusive sense) "Alice or Barbara is going to the party" (A v B), and we discovered it true that Alice is going and true that Barbara is going, then we would know the above statement (A v B) to be true. The statement claimed that one or the other, possibly both, are going, and both are going. Similarly, if only one of them is going, the statement would still be true. The only time the disjunction would be false would be when neither are going. This, then, will be our rule for disjunction. An *or* statement (A v B) is true when either one or both of the simple statement parts are true, and false only when both parts are false. Accordingly, this is how we picture this rule symbolically, along with its representation using variables:

Table (rule) for disjunction (v)

A	B	A v B	p	q	p v q
T	T	T	T	T	T
T	F	T	T	F	T
F	T	T	F	T	T
F	F	F	F	F	F

We can now use all three of our rules presented thus far to make one symbolic picture or table.

Truth Table for *not* (~), *and* (•), and *or* (v)

p	q	~p	~q	p•q	p v q
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	F	F

Before proceeding, be sure to understand how to read this summary table. The p and q columns on the left side merely show the initial possibilities and combinations of T and F. Then any given T or F under a (~), (•), or (v) column shows the result. So when p = T in the first row, ~p = F. When p and q are T, F respectfully (2nd row) the (•) column show the result is F and the (v) column shows the result is T. A computer thinks this way. If we asked a computer to compute A • B given that A = F and B = F, as a mechanical device it would not really think. It would look for the F, F situation under the p and q columns (4th row), then scan over to the (•) column and see the result F.

This symbolic picture shows what we eventually understand when we learn to use abstract words correctly. We know how a computer represents and processes these concepts. Exactly how our brains or minds represent these concepts is a matter of much debate in philosophy. From a practical point of view, the value of this table is that in using it we can mechanically discover the truth value of a statement if we know the truth value of its component parts. We can compute more complicated statements by analyzing their simple parts. For instance, to take a not too complicated statement first, if we had the following statement, "We will hire either Johnson or Kaneshiro, but not both," and we discovered it is true that Johnson will be hired (J = T) and false that Kaneshiro (K = F) will be hired, then we can mechanically conclude that the statement is true in the following way:

Translation:	(J	v	K)	•	~(J	•	K)	J = T
Substitution:	(T	v	F)	•	~(T	•	F)	K = F
Result:	(T	v	F)	•	~(T	•	F)	
		T		•	~	F		
		T		•		T		
				T				

If we thought like a computer, all we would do is replace J with T and K with F and plug in the values (second row). Then we would mechanically compute the truth value by figuring out that (T v F) = T and (T • F) = F (fourth row). Then we would compute ~F = T (fifth row). And finally we would compute (T • T) = T (sixth row). How would a computer know (T v F) = T and (T • F) = F? We have to tell it by programming it with the Truth Table for *not*, *and*, and *or* above. It would look under the p and q column on the left hand side until it saw T, F for p and q (second row). Then it would move over to the (v) column in the second row and see T. Then it would go under the (•) column in the same row and see F. Every step it would make would be to mechanically follow what the p and q table above shows. For the last step it would look at the first row and see T, T for p and q. Then it would look under the (•) column and see T. It would not really be thinking unless of course we "think" the same way and this is all there is to thinking. Is our thinking just many computations of bits of information?

We could, of course, have known the truth value result of this statement without symbolic logic. Because the above statement is the expression of an exclusive *or* and we were told that Johnson will be hired but Kaneshiro will not, we know that the meaning of the exclusive disjunction is fulfilled (one or the other but not both) and the statement is true. However, the practical value of mapping our common sense in this way is not only to understand the process of analysis, but to be able to compute much more complicated expressions without getting lost. This mechanical method is useful for such complicated examples as the following:

"It is neither the case that the Chiefs and the Raiders will both not be participating in the playoffs nor the case that the Patriots will be participating, even though either the Chiefs will not be participating or the Raiders will not be participating."

Here is the translation:

~[(~C • ~R) v P] • (~C v ~R)

If we discovered that the Chiefs will be participating (C = T), the Raiders will not be participating (R = F), and the Patriots will be participating (P = T), then we can calculate that the above statement is false as follows:

Translation:		~[	(~C	•	~R)	v	P]	•	(~C	v	~R)		C = T
Substitution:		~[	(~T	•	~F)	v	T]	•	(~T	v	~F)		R = F
Result:	(1)	~[	(~T	•	~F)	v	T]	•	(~T	v	~F)		P = T
	(2)	~[	(F	•	T)	v	T]	•	(F	v	T)		
	(3)	~[		F		v	T]	•	(	T	)		
	(4)	~[				T	]	•	(	T	)		
	(5)					F		•		T			
	(6)							F					

At this stage do not worry about understanding this statement. Although statements such as this one are routinely made by NFL football announcers in the United States in the month of December, what is important is how a computer would arrive at false (F) in a mechanical neutral way.

Before we try some exercises, let's focus on some of the mechanical features of this truth value calculation. Notice that step (1) involves simply substituting the indicated truth values for C, R, and P. Next, a series of simple calculations are made using the Truth Table for (*not*), (*and*) and (*or*) above, following a principle of working from the "inside (of parentheses) out." So, if you were a computer, you would first scan Result line (1) and convert all ~Ts to Fs and all ~Fs to Ts, because this is what your program (Table above) tells you to do when you process a ~T or a ~F. This procedure gives us line (2). Next, the computer would analyze each piece of this line that involves a single connective, go to its program, and then transfer a result. For instance, the (F • T) piece in Result line (2) is converted to F in Result line (3), because the third row of the table above is the F-T possibility for p and q and under the • column the computer would see that the result is F. Likewise, the (F v T) piece on line (2) would be converted to T, because the computer would see that the F-T case (third row again) produces a T under the v column for this row. Next, the computer would convert the (F v T) piece in line (3) to T. Notice that nothing happens to the first negation until line (5). Not until the (F v T) piece in line (3) is converted, can the negation be applied. Not until we have a single T within the rackets in line (4) can the negation be applied, that is, ~[T] = F. Finally, the (F • T) in (5) is converted to F, the final result and the truth value for this statement, given the input of C = T, R = F, and P = T.

It is important to understand and get a feel for how apparent complexity can be broken down into simple "bits." Essentially, computers are bit-brains; they may work very fast but each step in their thinking is no more than answering "yes" or "no" to a question. Should electricity be allowed to go through a channel or not? This process is identical to what we are doing here: Does the bit we are faced with convert to a T or an F? In order to find the following exercises simple, you must think like a computer, staying calm, focused, and bit-oriented. Although we derived the table for *not*, *and*, and *or* above from our common sense, we can forget for now where this table came from and just follow it like a computer or a Komodo dragon. In other words, don't think! Your entire intellectual effort amounts to no more than looking at the table above and staring at the correct row and column combination -- to see that (T • F) = F, that (F v F) = F, that ~F = T, and so on.

Exercise I

If A, B, and C are true statements, and X, Y, and Z are false statements,
which of the following are true and which are false? Follow the format used above: Translation, Substitution, and Result.

1. (C v Z) • (Y v B)

2. (A • B) v (X •Y)

3. ~(B v X) • ~(Y v Z)

4. ~(C v B) v ~(~X • Y)

5. ~B v C

6. ~(B v X)

7. ~X v A

8.* ~(X v Y)

9. ~[(B v A) v (~A v B)]

10. ~[(~Y v Z) v (~Z v Y)]

11 . ~[(~C v Y) v (~Y v C)]

12.* ~[(~X v A) v (~A v X)]

13. ~[A v (B v C)] v [(A v B) v C]

14. ~[X v (Y v Z)] v [(X v Y) v Z]

15. [A • (B v C)] • ~[(A • B) v (A • C)]

16.* ~[X • (~A v Z)] v [(X • ~A) v (X • Z)]

17. ~{[(~A v B) • (~B v A)] • ~[(A • B) v (~A • B)]}

18. ~{[(~C v Z) • (~Z v C)] • ~[(C • Z) v (~C • ~Z)]}

19. [A v (B • C)] • ~[(A • B) v (A • C)]

20. [B v (~X • ~A)] • ~[(B v ~X) • (B v ~A)]

Logical Connectives Continued: If…then (⊃) *and If and only if* (≡)

Having now come to an understanding of the connectives (~), (•), and (v), we need to provide a symbolic picture for the usage of (⊃) and (≡) . We begin with (⊃), which recall from Chapter 7 is called a **conditional**.

Similar to disjunction, providing a symbolic picture of conditional statements is complicated by the fact that we find many different usages of the *if . . . then* phrase in our language. We have seen that necessary and sufficient conditions are translated as conditionals, as well as *only ifs* and straightforward *if . . . thens*. So, let's see if we can come to an intuitive understanding of the core meaning of a conditional by thinking about the following sentence:

"If Friday is a nice day, then I am going to the beach (you will not have to come to class)." $(N \supset B)$

Suppose I made this unexpected statement on Wednesday. Suppose that, in spite of how unorthodox just taking off on a regular class day would be for a college professor, on Friday it is a nice day and I go to the beach. We would agree that my statement is now true. I said that if Friday was a nice day I would go to the beach, and it was a nice day and I did go to the beach. $(T \supset T) = T$

Now, suppose on Friday the weather is beautiful, but instead of going to the beach I come to class and give a surprise quiz! It is now obvious that my statement is false and that any attempt on my part to spin and claim the statement had become "inoperative" would be met with the justifiable contention that my statement is just plain false. $(T \supset F) = F$

To summarize what we have agreed upon so far, we see that when the **antecedent** (that part of the statement that follows *if* and translated on the left side of the ($\supset$) symbol) is true and the **consequent** (that part of the statement that follows *then* and translated on the right side of the ($\supset$) symbol) is true, then a conditional statement is true. When the antecedent is true, but the consequent is false, then the conditional statement is definitely false.

There are two remaining possible combinations to cover. Suppose now that it is false that the weather is nice on Friday and also false that I go to the beach. I had claimed that if the weather was nice, then I would go to the beach. The weather is not nice and I do not go to the beach. Clearly, my statement is not false and I am not lying as in the previous case. My action is not inconsistent with what I claimed I would do, so we conclude the statement is true. When we think about what a conditional statement claims, it is not surprising to see that a conditional will be true when both the antecedent and the consequent are false. $(F \supset F) = T$

Finally, we come to the most puzzling case. Suppose it is false that the weather is nice, but I go to the beach anyway. I said that ***if*** the weather was nice, then I would go to the beach. I did not say what I would do if the weather was bad. By going to the beach anyway I did not lie, nor have I acted in any way inconsistent with my statement. It is invalid to infer from my going to the beach that the weather is nice. In using a conditional statement, we are thinking hypothetically, not categorically. A conditional statement does not assert that its antecedent is true, but only that **if the antecedent is true, the consequent is true** also. Furthermore, a conditional statement does not assert that its consequent is true, but only that **the consequent is true, if its antecedent is true**. Hence, we cannot say my statement is false, if the antecedent is not true. So, we conclude that my statement is true. This then will be our full rule for ($\supset$) in propositional logic: **A conditional statement is false only when the antecedent is true and the consequent is false, and true otherwise**.

Here is how we will picture this rule symbolically:

Table (rule) for if. . .then (⊃)

N	B	N ⊃ B	p	q	p ⊃ q
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	F	F	T

The T that we put for the third possibility is admittedly a "shaky" true.[1] It reveals that we are capturing only a partial meaning common to the use of conditional statements. If we mistakenly thought that this interpretation represented the absolute meaning of all conditional statements, we would be forced to declare as true such statements as "If I sneeze within the next five minutes, then the whole world will disappear" -- provided I don't sneeze within the next five minutes! If I don't sneeze within the next five minutes, then the antecedent is false and, based on the above interpretation, the statement is true. Worse, the statement "If I don't sneeze within the next five minutes, then the world will disappear" would also be true – provided I do sneeze within the next five minutes. Worse yet, it follows that the world will disappear (D) within the next five minutes whether I sneeze (S) or not.

The argument:

1. S ⊃ D
2. ~S ⊃ D /∴ D   is valid!

This result shows that not only are there symbolizations and interpretations of meaning that are parts of more advanced logics (we will cover some examples in Chapters 11 and 12), but that there will always be some "holes" in any attempt to completely capture the world with logic. As has been emphasized repeatedly in this book, logic is simply a set of practical tools that can be used when appropriate to provide clarity in a complex situation, to follow a disciplined reasoning trail that will help us test our beliefs, and to organize our thoughts for decision making. Logic is a set of human tools, and the fact that these tools are useful in many situations should not make us so overconfident that we believe that we are somehow capturing the essence of life or of reality when we are logical. Some modern philosophers believe that past philosophical mistakes were made due to this overconfidence. In Western culture, many historical figures believed that in

[1] The controversy about the third row has a fancy philosophical name. It is called the problem of counterfactual conditionals. If you are interested, do an Internet search on the concept. I once had a former student call me up about 2am. He was now in graduate school working on his Ph.D. in economics. He said something like, "I know I should have paid more attention at the time, but can you go over that problem of counterfactual conditionals again?"

practicing logic and mathematics they were reading the mind of God. This in turn led to the belief that if one was logical or rational, one was automatically morally superior and had a divine sanction to "educate" and control people of a different culture.

Some twenty-first-century philosophers and logicians accept the admonishment of the philosopher and scientist Pierre Duhem, that "Pure logic is not the only rule of our judgments." For Duhem, human beings are also capable of judgments of "good sense" that are necessarily less rigorous and more vague than the sharp knife of deductive logic. As we saw in Chapter 3, such causal implication claims (claims regarding causal connections) as to the relationship between sneezing and the state of the world or between cigarette smoking and lung cancer must be appraised using the less precise standards of inductive inference. Applying such standards to the above examples, we can say with the relative confidence of good inductive sense that such claims regarding a connection between sneezing and the state of the world are false. All our background knowledge, all our scientific theories about the physical world, and all our historical experience lead us to believe that there is no connection between sneezing and the world disappearing. As with the connection between cigarette smoking and lung cancer we could be wrong—it may be that the world works in such a way that there is no connection between sneezing and the world disappearing until ten minutes from now!—but based on the evidence it is very unlikely that we are wrong. Thus, the F that we put for *if. . . then* statements regarding sneezing and the state of the world may not be as categorical and as certain as the F we put for "If Friday is a nice day, then I am going to the beach" when the antecedent is true and the consequent is false, but our commitment to the false nature of these statements is equally rational.

There is a joke: "Just because you're paranoid doesn't mean they AREN'T after you." True, but the possibility that someone is after me does not mean that it would be rational for me to believe that they ARE after me. As a tool, the rules of propositional deductive logic have a particular range of applicability. But just as it would be stupid to use a carpenter 's hammer during a medical operation or a surgeon 's scalpel to cut a piece of wood, so it would be inappropriate to use the rules of propositional logic for all situations. We will see an example of the possible need for more refined logical tools in Chapter 12.

Finally, to define *if and only if* ($\equiv$), a **biconditional**, lets follow up on the discussion from Chapter 7 on the statement:

"You go out tonight if and only if you clean your room." $O \equiv R$

We could use our commonsense intuition, but because few people use this phrase much in daily life, a better way is to show the power of what we have accomplished thus far. Because a statement such as "You go out tonight if and only if you clean your room" is actually a combination of two conditionals -- "You go out tonight **only if** you clean your room" ($O \supset R$) **and** "**If** you clean your room, then you go out" ($R \supset O$) -- we can figure out what a biconditional means by analyzing it in terms of what we have already learned about conditionals and conjunctions . We simply have to put together our definitions of

($\supset$) and (•) to figure out the statement ($O \supset R$) • ($R \supset O$), which is an equivalent way of saying $O \equiv R$. We can do this by making a table as follows.

O	R	$O \supset R$	$R \supset O$	$(O \supset R) \bullet (R \supset O)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

To summarize : We defined ($O \supset R$) and ($R \supset O$) according to the ($\supset$) rule and then put them together by the (•) rule. This then will be our rule for ($\equiv$): **A biconditional statement ($O \equiv R$) is true only when both components have the same truth value and false otherwise**.

At this point, a little commonsense reflection shows why this is so. When a mother tells her son that he will go out if and only if he cleans his room, the mother's statement would be true in only two cases: when the son cleans his room and the mother fulfills her promise by letting the son go out (row 1 above), and when the son does not clean his room and suffers the consequence of not going out (row 4 above). In the other two cases—when the son does not clean his room and the mother lets him go out anyway (row 2 above), and when the mother does not let her son go out even after he has cleaned his room (row (3) above)—the mother's statement would be false. Further reflection on this case reveals how understanding the difference between *only if* and *if and only if*, and the related difference between a necessary condition, a sufficient condition, and a necessary and sufficient condition is crucial in many communication and negotiation situations, not only in family relationships, but in public and international relations as well.

As we saw in Chapter 7, if a mother were to say to her son "You go out tonight only if you clean your room" ($O \supset R$), a son trained in logic would negotiate for a better condition! He would know that his mother only specified a necessary condition for going out, and that his mother would not be lying if after he cleaned his room, his mother then asked him to also do the dishes. The son would want to negotiate for an *if and only if* rather than just an *only if*. In this way, it would be sufficient for him to clean his room and, assuming his mother is consistent, he would go out. Similarly, when Iran held U.S. hostages in the late 1970s, statements from Iranian officials such as "We will release the hostages only if you release Iranian assets held in U.S. banks" ($H \supset A$) would not have been acceptable to United States officials. Like the son's case, the United States would want to negotiate an *if and only if*.

All of this may be a little too much for you to digest at this point, and we will be going over the logical implications similar to this imbedded in our language much more thoroughly later. But everything that was said in the last paragraph can be summarized in a simple symbolic picture. Like the old adage that a picture is worth a thousand words, symbolic logic attempts to summarize complicated linguistic relationships, just as mathematics attempts to summarize complicated numerical relationships. So, to make a long story short, here is our symbolic picture (our final rule) for a biconditional, a picture that summarizes hundreds of words in the previous paragraph.

Table (rule) for if and only if (≡)

p	q	p ≡ q
T	T	T
T	F	F
F	T	F
F	F	T

We can now summarize all our rules again in one table using only variables, reflecting the important point that these rules apply to any statements whatsoever.

Final Truth Table for Logical Connectives:

p	q	~p	~q	p • q	p v q	p ⊃ q	p ≡ q
T	**T**	**F**	**F**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**F**	**T**	**F**	**F**
F	**T**	**T**	**F**	**F**	**T**	**T**	**F**
F	**F**	**T**	**T**	**F**	**F**	**T**	**T**

Exercise II:

Repeat the procedure as in the previous exercise: A, B, C = true; X, Y, Z = false. Use the final table above to determine if each statement below is T or F.

1. A ⊃ (B ⊃ C)

2. A⊃ (B ⊃ Z)

3. A ⊃ (Y ⊃ C)

4. A ⊃ (Y ⊃ Z)

5. X ⊃ (B ⊃ C)

6. X ⊃ (B ⊃ Z)

7. X ⊃ (Y ⊃ C)

8. X ⊃ (Y ⊃ Z)

9. (A ⊃ B) ⊃ Z

10. (X ⊃Y) ⊃ Z

11.* [(X ⊃ Y) ⊃ B] ⊃ Z

12. [(B ⊃ Z) ⊃ B] ⊃ Z

13. [(X ⊃ A) ⊃ X] ⊃ X

14. [X ⊃ (Y v Z)] ⊃ [(X ⊃ Y) ⊃ Z]

15. {[A ⊃ (B ⊃ C)] ≡ ~X} ⊃ {X ⊃ [(A • B) ⊃C]}

16.* [(A ⊃ Z) • (Z ⊃ A)] ⊃ ~[(A • Z) v (~A v ~Z)]

17. {[X ⊃ (Y ⊃ Z)] ⊃ [(X • Y) ⊃ Z]} ≡ [(X ⊃ A) ⊃ (B ⊃Y)]

18. (B ⊃ A) ≡ (~A ⊃ ~B)

19. ~(A • B) ≡ (~A v ~B)

20. [A ⊃ (B ⊃ C)] ≡ [(A • B) ⊃ C]

Shortcuts and Human Learning

Before we continue, it is worth reflecting on the different effects doing the above exercises would have on a simple computer and the average human being. If you did all forty of the above exercises, you probably noticed some shortcuts after awhile. For instance, a close look at the truth table result for (•) shows that any time a part of a conjunction is false the entire statement is false. So, if we had the computation

F • {[~X ⊃ (~Y ⊃ Z)] ⊃ [(X • Y) ⊃ ~Z]}

we instantly know that the result is false without having to do any computations for the connectives inside the braces on the right hand side of the (•) statement. Similarly, because a disjunction is true when any part of the statement is true, we would know instantly that

T v {[~X ⊃ (Y ⊃ ~Z)] ⊃ [(X • ~Y) ⊃ Z]} is true without having to do any other computations on the right hand side of the (v) statement.

For conditional statements there are two shortcuts. Because the only time a conditional is false is when the antecedent is true and the consequent is false (T ⊃ F), whenever the antecedent is false or the consequent is true, a conditional is true. At a glance we can see that

F ⊃{[~X ⊃ (Y ⊃ Z)] ⊃ [(X • Y) ⊃ Z]}

and

{[X ⊃ (~Y ⊃ ~Z)] ⊃ [(X • Y) ⊃Z]} ⊃ **T**

are both true.

You may not have noticed these shortcuts, but you would have if you were forced to do hundreds of exercises like the above. Your mind would naturally get bored being a Komodo-dragon-like computer that must do every single computation, step by step. After awhile you would be able to just glance at a problem like 15 in Exercise II and see that the statement is true. Because the antecedent X within the major consequent is false, you would be able to quickly see that

15. {[A ⊃ (B ⊃ C)] ≡ ~X} ⊃ {F ⊃ [(A • B) ⊃ C]}
{[A ⊃ (B ⊃ C)] ≡ ~X} ⊃ **T**
T

A simple computer programmed to follow our final truth table above would not learn these shortcuts. One of the goals of present-day computer science and efforts in what is called "artificial intelligence," or AI, is to see if more sophisticated and powerful computers can be constructed that can "learn" higher-order connections on their own without human beings programming them to specifically see the shortcuts. The fact that a simple computer that thinks in terms of step-by-step bits cannot learn connections on its own may mean that the human brain is not just a bigger or more complex bit brain, as was once thought by some computer scientists. It may mean that the human brain works in terms of networking systemic connections and patterns.

Truth Tables, Validity, and Logical Pictures

We are now ready to learn our first symbolic method for analyzing *arguments* to be valid or invalid. To accomplish this task, you need to know two things: (1) You need to be proficient in the use of the symbolic picture/table of our commonsense understanding of the logical connectives of our Final Truth Table (**FTT**) above; and, (2) most important, you need to remember what a valid argument is—you must understand the concept of deductive validity covered in Chapter 1. Many students become quite good using the **FTT**, but many also do not know what they are doing because they are still struggling with all the ramifications of the concepts of validity and invalidity. Luckily, the truth table method that we will be learning in this section will offer not only a technique for analyzing arguments to be valid or invalid, but a pictorial way of reviewing and reinforcing the concepts of validity and invalidity. The one-picture-is-worth-a-thousand-words adage will again be applicable. With a few simple truth tables we will be able to summarize all the basic points made in Chapter 1.

Let's review. Recall that the technical definition of a **valid deductive argument** given in Chapter 1 was **an argument that does not allow for the possibility of true premises and a false conclusion**. The key word in this definition for us now is *possibility*. Recall how difficult it was in some of the Chapter 1 exercises to see if it was possible, given the truth of the premises, not to be locked into the truth of the conclusion. A truth table eliminates this uncertainty by painting a picture of an argument's total logical possibilities, showing us directly whether a given argument allows for true premises and a false conclusion or not.

To illustrate this, let's start with a simple argument that we know is valid, an argument in which our common sense is sufficient for seeing that we are locked into the conclusion. Suppose a history teacher told John just before the final exam that he was guaranteed to pass the course (C), if he passed the final exam (P). Suppose John then passed the final exam. Taken as premises, if this information were true, we would be guaranteed that John passed the course. We would have a valid argument as follows:

1. P $\supset$ C
2. P   /$\therefore$ C

Please recall from C7 that /$\therefore$ indicates the conclusion. So we have two premises and a conclusion. A truth table proof that this argument is valid looks like this:

Table T1

			p1		p2	c
P	C	P	$\supset$	C	P	C
T	T		T		T	T
T	F		F		T	F
F	T		T		F	T
F	F		T		F	F

Constructing this table involved a number of simple steps.

1. Decide on the possible logical combinations of true and false, given the number of simple statements. Because we have two simple statements (P, C), we have

four possible combinations. The two columns on the left side show the combinations.

P could be true and C could be true (row 1). P could be true and C could be false (row 2). P could be false and C could be true (row 3). And P and C could both be false (row 4).

2. Make a column for each premise and the conclusion. Notice that each premise is indicated by p1 and p2 and the conclusion is noted by c.

3. Figure out the truth value for each row in each premise and conclusion column by using the given possibilities decided in 1 and the **FTT**. (For instance, row 2 under the premise $P \supset C$ is false because P is true in that row and C is false, and our **FTT** dictates that when the antecedent of a conditional is true and the consequent is false, the conditional is false. $(T \supset F) = F$.

4. After completion of the table, look across each row under the premises and conclusion and see if any row has all true premises and a false conclusion. If not, the argument is valid. If there is at least one case of all true premises and a false conclusion, then the argument is invalid.

In looking at the table we have constructed for the above argument, we see that no row contains a case of all true premises and a false conclusion. This shows that this argument will not allow (given any possibility) for true premises and a false conclusion. Thus, it is a valid argument. However, it is important to understand and remember that valid arguments will allow for false premises (rows 2, 3, and 4) and false conclusions (rows 2 and 4). Also note that if the conclusion of a valid argument is false, at least one of the premises is false (rows 2 and 4). Remember that judging an argument to be valid is not to judge the specific content of an argument, but only the reasoning trail. So, if John did not pass the course (rows 2 and 4), we would know only that either his teacher did not tell him the truth (row 2) or John did not pass the final (row 4). But this possible content outcome would not change the validity of the reasoning.

Understand that row 1 by itself (all true premises and a true conclusion) does not show validity. As we will see shortly, and as was emphasized in Chapter 1, invalid arguments can also have true premises and a true conclusion. What shows the argument above to be valid is that inspection of all the rows reveals no case with all true premises and a false conclusion. The first row contains the ideal case of a sound argument, but only because we know the argument is valid. Validity must be determined before soundness can be determined.

To reinforce these points, let's look at a truth table of an invalid argument. Suppose our history teacher made the same statement to the effect that if John passes the final exam, he would pass the course. Suppose John's friend finds out that he did not pass the final exam, and infers from this that John must not have passed the course. This would be an invalid inference, because the history teacher did not say that passing the final exam was the only way that John could pass the course. The history teacher said that if he passed the final exam, he would be guaranteed of passing the course—which left open the possibility that there might be other ways of passing the course. He might have failed the final exam, but obtained a sufficiently high score on the final that, combined with his other grades (other exams and quizzes), he just made a passing score for the course. Or, an extra credit option might have allowed him to get enough points to pass the course. Neither of these possibilities is inconsistent with the history teacher's statement to John. The history teacher was telling John what would guarantee passing the course, not telling him the only way to pass the course.

To show that this argument is invalid and to reinforce the conceptual difference with a symbolic picture between validity and invalidity, here is a truth table for this invalid inference.

Table T2

1. P ⊃ C
2. ~P /∴ ~C

			p1		p2	c	
P	C	P	⊃	C	~P	~C	
T	T		T		F	F	
T	F		F		F	T	
F	T		**T**		**T**	**F**	***
F	F		T		T	T	

Notice the third row. Unlike our first argument, this argument does allow for a possible case of all true premises and a false conclusion. It also allows for all true premises and a true conclusion (row 4). This table underscores pictorially the important point made about invalid inferences in Chapter 1. Valid inferences are preferred over invalid ones not because valid arguments always have true conclusions or because invalid arguments always have false conclusions, but because when we have all true premises and a valid argument we are guaranteed a true conclusion. Whereas even if we have all true premises in an invalid argument, we are guaranteed nothing, the conclusion could be true or false. (Note the 3rd and 4th rows.) We want our effort in finding reliable premises and thinking about what follows from them to count. If our premises are true and the reasoning valid, then we are guaranteed of learning something in the conclusion. Even if we find the conclusion of a valid argument to be false, we still learn something important -- we learn that at least one of our premises is false. Remember, we test our beliefs with valid reasoning. With invalid reasoning we learn nothing; we do not test our premises when it is discovered that the conclusion is false. Even when the conclusion is false, the premises can still be all true.

The above cases are models for constructing a truth table for any argument. However, a truth table would not have been needed in either case -- our common sense would have been sufficient for judging these arguments. A truth table serves a more useful purpose for arguments such as the following:

> **If we pay our medical bills and buy the new car, then we will not have enough money for basic necessities. We have to pay our medical bills. So, if we don't buy the car, then we will have enough money for basic necessities. (M, C, B)**

Perhaps you can tell that something does not seem quite right with the conclusion of this argument. But are you sure it is an invalid argument? To be sure, let's do a truth table. First, the translation:

1. (M • C) ⊃ ~B
2. M /∴ ~C ⊃ B

Note that in following the steps for truth table construction listed above, the first step of determining all of the logically possible combinations given M, C, and B will now be more complicated. There are now three simple statements that could be true or false, so our table will have to be longer and involve more rows down. We could use our common sense to do this: We could start with M, C, and B all true, then M and C true, and B false, and so on until we ended with M, C, and B all false. However, you should be able to see that in very complicated arguments in which four, five, or six or more simple statements could be true or false, figuring out the possible combinations would be a nightmare. It would be difficult to keep track of which combinations we already had and which ones we still required to cover all the logical combinations.

It is in cases such as this that mathematics comes to our rescue. As noted earlier, mathematics has been built up through the centuries to make life easier for us. In our situation, mathematicians have discovered a little equation that will enable us to mechanically and effortlessly figure out the logical combinations given any number of simple statements: $2^y = n$. Like all mathematical equations, it looks intimidating until you are told what it means. To use it, you need only be able to multiply by two. The ***y*** stands for the number of simple statements, and the ***n*** stands for the number of rows down needed to cover all the logical combinations of true and false, given ***y*** (the number of simple statements). In our case, we have three simple statements. Plugging this into the equation, we have $2^3 = 8$. ($2^3 = 2 \times 2 \times 2 = 8$) Think how much time this equation would save if we had a truth table with six simple statements! Using the equation we would know instantly that the truth table would need sixty-four lines down to cover all the logical possibilities. It would probably take at least several hours to figure this out with our common sense, and then more time to make sure that we did not make a mistake in repeating a line of possibilities or omitting one.

However, this equation does not tell us how to arrange the various Ts and Fs; it only tells us with certainty how many rows down we need to cover all the possibilities. But arranging the Ts and Fs is also simple and mechanical. We make the first column (the first simple statement), in our case M, half Ts and half Fs. So, if we have eight lines down, the first four rows will be true and the second four rows will be false. Then we take the next column, in our case C, and make it half (in a sense) of what M was. So, because M was four Ts and four Fs, C will be two Ts and two Fs all the way down. The next column, B, will then be half of C's distribution, one T and one F all the way down. Here is what this will look like symbolically.

Three simple statements = 8 rows. So,

M = 4 Ts and 4 Fs
C = 2 Ts and 2 Fs
B = 1 T and 1 F.

M	C	B
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Note the elegance of this process (mathematicians and logicians love elegant processes). Without thinking very much, we have covered all the logical combinations of true and false for three simple statements. The first row has all true and the last row has all false, and between these two rows every other combination is covered. This combination of simple equation and assignment of truth values will work for any number of simple statements. If we had four simple statements, we would need sixteen rows. The first simple statement would be assigned eight Ts and eight Fs, the second simple statement would be assigned four Ts and four Fs all the way down, the third simple statement would be assigned two Ts and two Fs all the way down, and finally, the last simple statement would be assigned alternating Ts and Fs all the way down. As in the case of three simple statements, we would have all Ts in the first row and all Fs in the last row, and every other combination in between. (Take a break here and try this. For the letters G, X, Z, and A (see #5, Ex III), set up a table of logical combinations of true and false to verify what I have just said.)

We can now finish steps 2 through 4 listed above for constructing a complete truth table as follows.

Table T3

Invalid: At least one row with all true premises and a false conclusion

						p1		p2		c		
M	C	B	(M	•	C)	⊃	~B	M	~C	⊃	B	
T	T	T				F		T		T		
T	T	F				T		T		T		
T	F	T				T		T		T		
T	F	F				**T**		**T**		**F**		***
F	T	T				T		F		T		
F	T	F				T		F		T		
F	F	T				T		F		T		
F	F	F				T		F		F		

Keep in mind the mechanics of how we arrived at the columns under p1, p2, and c. All we are doing is applying what we learned earlier for doing Exercises I and II. For instance, in the first row M, C, and B are all true. Hence for the first premise we have:

(M	•	C)	⊃	~B
(T	•	T)	⊃	~T
(T	•	T)	⊃	F
	T		⊃	F
			F	

Hence, F is placed under the p1 column in the first row. For the conclusion in the first row we have:

~C	⊃	B
~T	⊃	T
F	⊃	T
	T	

So T is put under the c column in the first row.

Notice that when we finish computing each row, this argument allows for all true premises and a false conclusion in the fourth row, verifying the suspicion that the conclusion seemed strange. What should have been concluded from the above premises is that a new car should not be purchased, if enough money is to be had for basic necessities (B ⊃ ~C), or that buying a new car will guarantee not having enough money for basic necessities (C ⊃ ~B). To say that buying the new car will guarantee not having enough money for basic necessities is not the same as saying that not buying the new car will guarantee having enough money for basic necessities. To show that this is so, and to illustrate a more piecemeal method for constructing more complicated truth tables, let's show that concluding B ⊃ ~C would give us a valid argument as follows.

Table T4

Valid: No row with all true premises and a false conclusion

						p1		p2		c	
M	C	B	(M	•	C)	⊃	~B	M	B	⊃	~C
T	T	T		T		**F**	F	**T**	T	**F**	F
T	T	F		T		**T**	T	**T**	F	**T**	F
T	F	T		F		**T**	F	**T**	T	**T**	T
T	F	F		F		**T**	T	**T**	F	**T**	T
F	T	T		F		**T**	F	**F**	T	**F**	F
F	T	F		F		**T**	T	**F**	F	**T**	F
F	F	T		F		**T**	F	**F**	T	**T**	T
F	F	F		F		**T**	T	**F**	F	**T**	T

Using this more piecemeal method, in constructing this table the first premise and the conclusion are broken down into parts and each part is given a column. The parts are then combined using the major connective to arrive at the final truth value. For instance, in premise 1 in the first row, the antecedent M • C is determined to be true and the consequent ~B is determined to be false. These results are then combined by (⊃) to get false. In row 1 of the conclusion, B is true and ~C is false. This is combined by (⊃) to determine false. The remaining rows are determined in a similar fashion. The value of this more piecemeal approach is to do everything step by step. The disadvantage is that when finished there are so many columns of true and false combinations, that it would be easy to get our eyes crossed and not look at the right results to determine if the argument allows for any case with all true premises and a false conclusion. To remedy this, the key results are in boldface print and we make sure that a p1 (for premise 1), p2 (for premise 2), and c (for the conclusion) are placed above the key column results. Upon inspecting just these columns, we see that this argument does not allow for any row with all true premises and a false conclusion. In the only rows that have a false conclusion (rows 1 and 5), at least one of the premises is false in these rows. Hence, the argument is valid.

Now it is time for you to construct your own truth tables. Use the four examples above as models to do the following exercises.

Exercise III

Construct a truth table for each of the following arguments, proving that each is either valid or invalid. If the argument is invalid, circle the key row that shows all true premises and a false conclusion. Do not circle any row if the argument shows valid. Do you know why?

#1
1. $A \supset B$
2. B $/\therefore$ A

#2
1. $A \supset B$
2. $C \supset B$ $/\therefore$ $A \supset C$

*#3
1. $(A \vee \sim B) \supset \sim C$
2. $A \equiv B$ $/\therefore$ $\sim C$

#4
1. $X \supset (\sim Y \supset Z)$
2. $Y \vee X$
3. $\sim Y$ $/\therefore$ $Z \bullet \sim Y$

#5
1. $(G \supset X) \bullet (Z \supset A)$
2. $\sim X \vee \sim A$ $/\therefore$ $G \equiv \sim Z$

Exercise IV

First translate each of the following arguments into symbolic notation. After checking with your instructor for the correct translations, construct truth tables and prove validity or invalidity. (Repeat: Don't do truth tables for these exercises until you are sure you have a correct translation!)

1. This man is either intoxicated or has diabetes. We know he does have diabetes. So, he must not be intoxicated. (I = The man is intoxicated; D = The man has diabetes; to provide the correct translation you must decide whether the *or* in the first premise is *inclusive* or *exclusive*.)

2. My wife's phone at work is busy. John's phone is also busy. If they are talking to each other, then both phones are busy. Therefore, my wife and John must be conversing over the phone. (W = My wife's phone at work is busy; J = John's phone at work is busy; T = My wife and John must be talking to each other.)

3. If God exists, then He is all-powerful. If He exists and is all powerful, then the universe is elegantly ordered. Because the universe is elegantly ordered, this shows that God exists. (G = God exists; P = God is all-powerful; U = The universe is elegantly ordered.)

4. Only if Jesus was more than a man are Christians right that he spoke the final truth about the meaning of life. Jesus rose from the dead, and this is sufficient to know that he was more than a man. This proves that Christians are right that Jesus spoke the final truth about the meaning of life. (M = Jesus was more than a man; S = Christians are right that Jesus spoke the final truth about the meaning of life; R = Jesus rose from the dead.)

5. If I don't qualify for the job, then you don't qualify; whereas, if you don't qualify, then I don't qualify. If both of us don't qualify, then Renee will get the job. So, if either of us does not qualify, Renee will get the job. (I = I qualify; Y = You qualify; R = Renee will get the job.)

6. *An old Volkswagen ad featured a photo of a street of identical houses. In front of each home is parked a red and white Volkswagen station wagon. Under the photo is this message (argument): **If the world looked like this, and you wanted to buy a car that sticks out a little, you wouldn't buy a Volkswagen station wagon. But in case you haven't noticed, the world does not look like this. So, if you want a car that sticks out a little, you know just what to do**.

   Identify the premises and conclusion of this argument and then use W = The world looks like this; S = You want a car that sticks out a little; V = You will buy a Volkswagen station wagon. (Assume that the phrase "you know just what to do" means that you will buy a Volkswagen station wagon.)

7. A basketball player named Johnson is suspended and fined for not showing the proper respect to his coach. His union asks for a ruling and an arbitration board rules that although the fine was justified, the suspension was not because Johnson was having "emotional problems" due to a marital separation. The coach responded that the ruling was inconsistent; that if Johnson was responsible for his action, then the fine and the suspension should be justified, and if he was not responsible, then neither should be justified.

   Below is a formalization of the coach's reasoning. Translate this argument:

   The view of the arbitration board that the fine was justified but the suspension was not is mistaken, for the following reasons: Either Johnson was or was not responsible . If he was, then the fine and the suspension were both justified. If he was not responsible, then neither the fine nor the suspension was justified. (R = Johnson was responsible; F = The fine was justified; S = The suspension was justified.)

8. If Clinton comes completely clean on the Whitewater investigation, then he must fire his White House counsel. If he evades the press on this issue, the Republicans will continue to gain political points against his presidency. If he comes completely clean on the Whitewater investigation, then he will not evade the press on this issue. So, if Clinton fires his White House counsel, the

Republicans will not continue to gain political points against his presidency.

C = Clinton comes completely clean on the Whitewater investigation.
F = Clinton fires his White House counsel.
E = Clinton evades the press on the Whitewater issue.
P = The Republicans will continue to gain political points against his presidency.

9. Often philosophers note that the concept of a reliable belief (Chapter 3) and the success of science are tied to a belief about how the natural world behaves, that there are uniformities in nature, that certain relationships are repeated and can be recognized if we pay attention. The argument below discusses these connections. Translate:

   It is not true both that human beings can make predictions about the future and not achieve reliable beliefs about how the natural world works. This is so, because if there are uniformities in nature, then human beings can achieve reliable beliefs about how the natural world works. Moreover, human beings can make predictions about the future only if there are uniformities of nature.

   P = Human beings can make predictions about the future.
   R = Human beings can achieve reliable beliefs about how the natural world works.
   U = There are uniformities in nature.

10. For scientific creationists, one of the grave implications of Darwin's theory of evolution is that there is no guarantee that human beings are special in the universe. They are very concerned about this, because they presuppose that it is not possible to have objective moral values and standards unless human beings are special. If this presupposition is true, then it would follow that if Darwin's theory is true it would not be possible to have objective moral values. But is this presupposition true? Analyze the following argument.

    If scientific creationism is true, then human beings are guaranteed to be special creatures in the universe and there is a source for objective values. If Darwin's theory of evolution is true, then human beings are not guaranteed to be special creatures in the universe. Because scientific creationism and Darwin's theory of evolution cannot both be true, it follows that there is no source for objective values unless human beings are special creatures in the universe.

    C = Scientific creationism is true.
    S = Human beings are guaranteed to be special creatures in the universe.
    O = There is a source for objective values.
    D = Darwin's theory of evolution is true.

11. In 2006, Vice President Dick Cheney's former chief of staff, Lewis "Scooter" Libby, testified that President Bush authorized him to disclose the contents of a highly classified intelligence assessment to the media to defend the Bush administration's decision to go to war with Iraq. Given that previously the Bush administration had vigorously protested that leaking classified intelligence information was illegal and harmful to the security of the United States, this admission raised many eyebrows. Could it be that this leaking was permissible because it was leaked by the President? Analyze the following argument.

    If the president of the United States (according to the Bush administration) leaks a classified intelligence document to the press, then the intelligence document can be legally leaked to the press; moreover, if an intelligence document can be legally leaked to the press, then an intelligence document is not classified. This is so, for the following reasons: If an intelligence document is classified, then it cannot be legally leaked to the press. If the president of the United States (according to the Bush administration) leaks a classified intelligence document to the press, then an intelligent document is not classified. If an intelligence document is not classified, then it can be legally leaked to the press.

    P = The president of the United States (according to the Bush administration) leaks a classified intelligence document to the press.

    L = The intelligence document can be legally leaked to the press.

    C = An intelligence document is classified.

12. Sustained business growth after the 2009 recession is possible only if education becomes synonymous with life long learning, yet most schools do not know education should be for more than immediate employment. This is so because, sustained business growth after the 2009 recession is possible only if technological innovation continues. Technological innovation continues if and only if education becomes synonymous with life long learning. If technological innovation continues only if education becomes synonymous with life long learning, then most schools do not know education should be for more than immediate employment.

B = Sustained business growth after the 2009 recession is possible.

T = Technological innovation continues.

E = Education becomes synonymous with life long learning.

S = Most schools do know education should be for more than immediate employment. xxx

Argument Forms and Variables

Throughout this book we have emphasized the importance of concepts. Many concepts may be abstract, but our ability to abstract is responsible for our success on this planet as a species and also forms a large part of what it means to say that human beings possess "intelligence." When you understand a concept, you have a net, so to speak, to throw over, control, and understand countless details. When a child understands the concept of a chair, he or she understands how to use any of a number of objects that we call *chair*, no matter how they differ in individual details. By grasping concepts we are able to see the forest and not get lost within the trees. The most important part of your academic life, no matter what your major, should consist of learning concepts and their interconnections, not the details of dates, names, and events. The latter are easily forgotten but just as easily looked up in books, computer data bases and search engines on the Internet. Concepts save time and give you a lot of power. With insight into concepts apparent complexity is seen to be simplicity in disguise.

As we have seen, logicians take this abstracting ability of ours very seriously. They slow our thinking down and identify patterns of thought. In the chapters on informal fallacies we saw that once we identified the argument structures or forms of fallacies we had a pattern that could be filled in by any number of details and issues. In Chapter 1 we saw formal patterns for valid and invalid deductive reasoning. In the remaining part of this chapter, and especially in the chapters that follow, we will be taking this form-recognition process one step further by expanding considerably the applicability of the notion of a variable -- our use of **p'**s and **q'**s.

Consider the following argument:

Example A1

If we pay our medical bills and buy the new car, then we will not have enough money for basic necessities this month. We will pay our medical bills this month, but we are also going to buy the new car. So, we will not have enough money for basic necessities this month. (M, C, B)

The translation of this argument would be:

1. (M • C) ⊃ ~B
2. M • C **/∴** ~B

This argument is obviously valid but if we had to do a truth table, we would have to construct a time-consuming table of eight rows. There is a much faster way. We instantly recognize the validity of this argument without doing a truth table because it has a form or pattern of reasoning that is as common as the concept of a chair. Recall that whenever we assert a conditional statement, we are claiming that if the antecedent is true, then the consequent must be true. So, if we assert a conditional statement as part of an argument and then find out that the antecedent is true, we are locked into asserting that the consequent is true as a conclusion.

The above argument has the same form as:

Example A2

If John passes the final, he will pass the course. John passed the final. So, John will pass the course. (P, C)

1. P ⊃ C
2. P /∴ C

Just as we used the logical variables **p** and **q** to capture the essence of our use of logical connectives, just as we used these variables in our **FTT** above to represent any statements whatsoever, so we can use variables to capture argument forms. What both of the above arguments have in common is two premises, the first being a conditional, the second repeating the antecedent of the conditional, and the conclusion repeating the consequent of the conditional. We can picture this commonality with **the argument form**:

1. p ⊃ q
2. p /∴ q

We know intuitively that this is a valid argument, and a simple truth table of this argument form corroborates our intuition:

			p1		p2	c
p	q	p	⊃	q	p	q
T	T		T		T	T
T	F		F		T	F
F	T		T		F	T
F	F		T		F	F

Valid, no row with true premises and a false conclusion.

Thus, we do not need to construct a time-consuming truth table of Example Al, but only its much simpler argument form. In general, if a table is needed to judge the validity or invalidity of a complex argument, we can always construct a table of an argument's form. Plus, once we know that the argument form is valid or invalid, no truth table at all is needed. If a complex argument fits a valid form, we already know the argument is valid. If a complex argument fits an invalid form, we already know the argument is invalid. For instance, consider this argument.

Example A3

1. $\{[(A \supset B) \equiv (C \vee \sim B)] \bullet \sim D\} \supset (\sim E \vee \sim F)$
2. $\{[(A \supset B) \equiv (C \vee \sim B)] \bullet \sim D\}$ $/ \therefore$ $\sim E \vee \sim F$

See the pattern?

$p = \{[(A \supset B) \equiv (C \vee \sim B)] \bullet \sim D\}$
$q = (\sim E \vee \sim F)$

A truth table that ignores the argument form would require sixty-four lines! But if we look carefully we see that this argument is just another example of the same form identified above and so would require only four lines. Plus, no table is needed at all. The table is already done so to speak. Just look carefully and you should see that A3 fits this form again:

1. $p \supset q$
2. p $/ \therefore$ q

All three arguments (A1, A2, A3) are the same argument in disguise so to speak, so we know by just seeing the form behind the scenes (a valid form) that all three arguments are valid. Apparent complexity is often simplicity in disguise.

To practice this form recognition process, here are some more examples of arguments and their much simpler argument forms. (Don't worry knowing if these arguments are valid or invalid yet. We will work more on making these judgments in C9.)

Example A4

1. $(P \bullet C) \supset D$
2. $\sim D$ $/ \therefore$ $\sim(P \bullet C)$

1. $p \supset q$
2. $\sim q$ $/ \therefore$ $\sim p$

Example A5

1. $(H \vee \sim T) \supset (S \supset P)$
2. $\sim D \supset (S \supset P)$ $/ \therefore$ $(H \vee \sim T) \supset \sim D$

1. $p \supset q$
2. $r \supset q$ $/ \therefore$ $p \supset r$

Example A6

1. (A v B) ⊃ (~C • ~D)
2. (~C • ~D) ⊃ (H v ~T) /∴ (A v B) ⊃ (H v ~T)

1. p ⊃ q
2. q ⊃ r /∴ p ⊃ r

A truth table based on the argument form of Example A4 would require only four rows and show this argument to be valid. A truth table of Example A5 would require only eight rows and show this argument to be invalid. A truth table of Example A6 would also require only eight rows and show this argument to be valid.

Exercise V

Identify the argument form for each of the following arguments.

1.

P ⊃ (S v ~T)
~D ⊃ (S v ~T) /∴ P ⊃ ~D

2.

(P • T) ⊃ (S v ~Y)
P • T /∴ S v ~Y

3.

(X v Y) ⊃ H
~H /∴ ~(X v Y)

*4.

~(S • G) ⊃ (T ⊃ Y)
(T ⊃Y) ⊃ ~P /∴ ~(S • G) ⊃ ~P

5.

(A • B) v (Y ⊃ ~T)
~(A • B) /∴ Y ⊃ ~T

6.

[(A • B) ⊃ ~X] • [G ⊃ (S v T)]
(A • B) v G /∴ ~X v (S v T)

*7.

(N ⊃ T) • (S ⊃ Y) /∴ N ⊃ T

8.

C ≡ (B • D) /∴ [C ⊃ (B • D)] • [(B • D) ⊃ C]

9.

(A • B) ⊃ (H v Y)
~(A • B) /∴ ~(H v Y)

10.

(~A v ~B) ⊃ (S ≡ H)
S ≡ H   /∴ ~A v ~B

Brief Truth Tables

As we have seen, a concept that plays a major role in logic is validity. Thus, we would expect a considerable amount of saved effort in constructing truth tables by a thorough understanding of this concept. Recall that the technical definition of validity is an argument that does not allow for any possibility of true premises and a false conclusion. Most arguments involve conclusions that are ***contingent*** statements. That is, they are statements that are true or false depending on the state of affairs in the world and thus their truth table will show a mix of true and false results given the different rows of possibilities. We can take advantage of this by realizing that to judge an argument by the truth table method we need only to check the rows in which the conclusion is false. If we are always looking for cases of true premises and a false conclusion, then cases where the conclusion is true cannot show invalidity.

Consider the following truth table where the result of the conclusion is constructed first.

Table T5

						p1		p2		c		
A	B	C	(A	•	B)	⊃	~C	~A	B	⊃	C	
T	T	T								T		
T	T	F						F		F		
T	F	T								T		
T	F	F								T		
F	T	T								T		
F	T	F		F		**T**	T	**T**		**F**		***
F	F	T								T		
F	F	F								T		

The conclusion is a contingent statement; its truth table result shows a mix of true and false results that depend on what might be the case in the world. Only in the second row and the sixth row (B = T and C = F) will the conclusion be false. Thus, these are the only possible rows that could show the defective situation of having all true premises and a false conclusion. In working backward starting with the second row we quickly see that the second premise is false. We need go no further regarding this row, because we are looking for a situation with all true premises and a false conclusion. Although we have a false conclusion in this row, we can't have all true premises because the second premise is false. However, working backward from the conclusion in the sixth row we find that all the premises in this row are true. Hence, the argument is invalid. If we were to have done this row first, this would have been the only row necessary to construct! Because once we find a row with all true premises and a false conclusion, we know that the entire argument is weak; the argument does not guarantee a true conclusion whenever the premises are true.

To reinforce this shortcut insight, consider the truth table results for number 7 in Exercise IV. Your table could have looked like this:

Table T6

				p1			p2					p3				c			
R	F	S	R	v	~R	R	⊃	(F	•	S)	~R	⊃	~(F	v	S)	~	(F	•	~S)
T	T	T														**T**	T	F	F
T	T	F				T	**F**		F		F	**T**				**F**	T	T	T
T	F	T														**T**	F	F	F
T	F	F														**T**	F	F	T
F	T	T														**T**	T	F	F
F	T	F									T	**F**	F	T		**F**	T	T	T
F	F	T														**T**	F	F	F
F	F	F														**T**	F	F	T

Only the second and sixth rows show a false conclusion. Because these are the only rows we need to check, working backward in the second row we find that although the third premise is true (the antecedent ~R is false, so the conditional is automatically true), the second premise is false. Thus, we need go no further on this row; we have discovered that this row cannot show all true premises and a false conclusion. Checking the sixth row, we find that the third premise is false, so we need work no further on this row. Because these are the only rows that had a chance of showing all true premises and a false conclusion, and we discovered that the premises cannot be all true when the conclusion is false, we have demonstrated that this argument is valid: It does not allow for any possibility of all true premises and a false conclusion. Recognizing the full implications of the concept of validity as applied to truth tables saves us a considerable amount of unnecessary busywork. It is unnecessary to work out the details of the remaining rows.

Incidentally, your truth table result for the first premise (R v ~R) should have shown that this premise is not a contingent statement because it is never false; there is no row where this premise is false. Statements that are always true, regardless of the possibilities in the world, are called ***tautologies***. Such statements provide ultimate security, but at a price: They don't say anything significant about the world and any premise or combination of premises implies them. If I were to tell you that the color of my car is black or it is not black, I would be telling you the truth no matter what the color of my car, but I would not be conveying any information to you about the color of my car. Furthermore, tautologies are useless as conclusions, because they make any argument valid, regardless of the premises. The above statement about my car would follow (be the conclusion of a valid argument) given the premise, "The moon is made of green cheese." It would also follow validly from the premise, "The moon is not made of green cheese." Because arguments are invalid if and only if they allow for a case of all true premises and a false conclusion, an argument that never has a case of a false conclusion is valid by default.

A similar problem occurs with statements that are called ***contradictions***. Contradictions are statements that are always false regardless of the situation in the world. If the first premise of the above argument was ~(R v ~R), its truth table result would show F in every row. Given such a premise, an argument is also valid by default, regardless of the nature of the other premises or the conclusion. This is so because with a premise that is always false, it is impossible to have a case with all true premises -- a necessary condition for invalidity. So, just as any conceivable premise or set of premises imply a tautology, contradictions (as premises) imply any conceivable conclusion. With a contradiction as a premise, the statements "The moon is made of green cheese" and "The moon is not made of green cheese" could both be conclusions of valid arguments. Although Eastern philosophers have generally believed that contradictions are valuable in understanding the true nature of reality, Western philosophers have generally believed that contradictions should be avoided like the plague, not only because we wish to make true statements but also because they "prove" everything to be true.

So, arguments that involve contradictory premises, such as A and ~A, would be valid by default, but useless. It should be apparent from this why logicians are so concerned with consistency. If our beliefs are inconsistent, they cannot all be true. In addition, they imply not only what we want them to imply -- other beliefs that we have -- but also the beliefs we think are not true. If I believed that an unborn fetus is a human being and should be granted all the rights of citizenship, and I also believed that the fetus is not a human being and thus should not be granted the rights of citizenship, then my contradictory beliefs imply the conclusions of both positions on abortion (pro-choice and pro-life), as well as that the moon is made of green cheese! Contradictions don't get us anywhere in the belief reliability game, precisely because they get us everywhere, so to speak.

Thus, from a logical point of view a large part of personal growth involves becoming more rational by recognizing that each of us has many inconsistent beliefs, and then making some attempt to resolve the inconsistency. A large part of mature reflective thinking, of philosophical and logical analysis, involves looking for and resolving hidden contradictions. From a scientific point of view, our goal should be to have noncontradictory, contingent statements as beliefs and then investigate the world empirically to see which ones are true. Simplified, life and the goal of finding reliable beliefs can be seen as a combination of conceptual and empirical problems. If our concepts are not consistent, we have a conceptual problem. For instance, in the mid-1990s the Hubble telescope seemed to provide evidence that the universe was younger than the stars in some star clusters. Our best theory of how the universe began seemed conceptually inconsistent, because we had thought that all stars evolved after the universe began. On the other hand, if our beliefs are not consistent with what experience continues to tell us, we have an empirical problem. If I believe that the world should end at 12 :00 a.m., January 1, 2000, then I have an empirical problem if I am still experiencing the world going about its normal business on January 2, 2000. Recognizing and solving problems is an inevitable part of life. And logical analysis, both for recognizing hidden contradictions and for testing beliefs against experience, plays a major role.

To return now to truth-table analysis, by carrying out the validity shortcut insight further, we see that we don't even need to construct a table of possibilities at all! If an argument can only show invalidity when the conclusion is false, why not "make" the conclusion false and then see if the argument we are checking can consistently have true premises. If it can, we know the argument is invalid; if it cannot, we know the argument is valid. As an illustration of this, consider the following argument and brief proof of invalidity.

Table T7

1 . $A \supset B$

2. $C \supset B$ $/\therefore$ $A \supset C$

1.	A	$\supset$	B	A = T
	T		T	B = T
		T		C = F

2. $C \supset B$ $/\therefore$ $A \supset C$

	F		T		T		F	
		T				F		Invalid

We have shown this argument to be invalid, because we can make the conclusion false and then consistently make the premises all true. Here are the steps in the construction of brief truth tables.

1. Start with the conclusion. Make the conclusion false. (Because the above argument has a conditional for its conclusion, we assign a T to the antecedent and an F to the consequent.)

2. Substitute the assigned truth values consistently. Whatever truth assignments were made in the conclusion must also be made in the premises. (We are committed to A being T, so A must also be T in the first premise. C is F in the conclusion, so C must be F in the second premise.)

3. Try to make all the premises true. Attempt to assign uncommitted values in such a way to make the premises true. (Because C is F in the second premise, we are free to put either T or F for B to make this premise true. We choose T because we are not free to put an F for B in the first premise. B must be T in the first premise, and we are free to choose T because C is F in the second premise.)

When a result of true premises and a false conclusion is created, we have essentially isolated the row in a long table that would show the argument to be invalid. However, for valid arguments we will not be successful in making the premises all true when we make the conclusion false, because valid arguments will not allow for this; if the conclusion is false, at least one of the premises will be false. See what happens when we try to show the following argument to be invalid.

Table T8

1. A ⊃ B
2. B ⊃ C /∴ A ⊃ C

1.	A	⊃	B					A = T
	T		T					B = T
		T						C = F

2.	B	⊃	C	/∴	A	⊃	C	
	T		F		T		F	
		F				F		Not Invalid

Or we could try:

1.	A	⊃	B						A = T
	T		F						B = F
		F							C = F

2.	B	⊃	C	/∴	A	⊃	C		
	F		F		T		F		
		T				F			Not Invalid

In attempting to make the first premise true, we must assign T to B. But this makes the second premise false. If we started with the second premise and made B = F, this would make the second premise true, but it would then make the first premise false. In all valid arguments, there will be a similar flip-flop in the premises. In making one premise true, another premise will be made false. Each attempt by itself does not show the argument to be valid, because we must check all possibilities. Each attempt only shows that we were not successful in creating the case of all true premises and a false conclusion. However, because B must be either T or F, in combining both cases we have covered all the possibilities and have shown that this argument must have at least one premise false, if the conclusion is false. Hence, it is valid.

Before trying some exercises, it should be noted that as nice as this brief method is, there are limitations. The brief truth table method works best when there is only one way to make the conclusion false. If an argument has a conclusion in which there are several ways to make the conclusion false, it would be a hasty conclusion to assume validity, if in checking for true premises given only one way of making the conclusion false, not all the premises could be made true. Because an assessment can be made only after all the possibilities have been checked, each assignment that would make the conclusion false must be checked. It is possible that although one assignment of truth values that makes the conclusion false will not result in all true premises, the next assignment will.

So, the brief truth table method works best with single statements or their negations (A, ~A), or conditional or disjunctive statements as conclusions, because only one way exists of making such statements false. Because biconditional statements can be made false in two ways, and conjunctions can be made false in three ways, the brief truth table method is more cumbersome to apply given conclusions with these connectives. In these cases, it is often more convenient and less confusing to do a long table.

Exercise VI

Show the following arguments to be valid or invalid using the brief truth table method.

1.

(A v B) ⊃ C
C /∴ A

2.

$A \supset B$
$C \supset B$
$\sim D \supset B \quad / \therefore \; A \supset \sim D$

*3.

$(A \equiv B) \supset C$
$B \supset A$
$\sim C \quad / \therefore \; \sim A \vee B$

4.

$A \supset (B \vee D)$
A
$D \supset (S \bullet \sim Y) \quad / \therefore \; \sim B \supset \sim Y$

5.

$(D \bullet C) \supset (H \supset \sim Y)$
$P \supset (D \bullet C)$
Y
$(P \bullet H) \vee (D \bullet C) \quad / \therefore \; \sim(D \bullet C)$

Hint: Put this argument into its argument form first.

Answers to Starred Items

I.

8. True

~	(X	v	Y)
~	(F	v	F)
~	(	F	)
		T	

12. False

~[(~X	v	A)	v	(~A	v	X)]
~[(~F	v	T)	v	(~T	v	F)]
~[(T	v	T)	v	(F	v	F)]

~[	T	v	F	]
~[		T		]
		F		

16. True

~[X	•	(~A	v	Z)]	v	[(X	•	~A)	v	(X	•	Z)]
~[F	•	(~T	v	F)]	v	[(F	•	~T)	v	(F	•	F)]
~[F	•	(F	v	F)]	v	[(F	•	F)	v	(F	•	F)]
~[F	•		F	]	v	[	F		v		F	]
~[	F			]	v				F			
	T				v				F			
					T							

II.

11. False

[(X	⊃	Y)	⊃	B]	⊃	Z
[(F	⊃	F)	⊃	T]	⊃	F
[	T		⊃	T]	⊃	F
			T		⊃	F
					F	

16. True

[(A	⊃	Z)	•	(Z	⊃	A)]	⊃	~[(A	•	Z)	v	(~A	v	~Z)]
[(T	⊃	F)	•	(F	⊃	T)]	⊃	~[(T	•	F)	v	(~T	v	~F)]
[(T	⊃	F)	•	(F	⊃	T)]	⊃	~[(T	•	F)	v	(F	v	T)]
[	F		•		T	]	⊃	~[	F		v		T	]
			F				⊃	~[			T			]
			F				⊃				F			
							T							

III.

3. Valid

							p1				p2			c
A	B	C	|	(A	v	~B)	⊃	~C	|	A	≡	B	|	~C
T	T	T		T	T	F	**F**	F			**T**			**F**
T	T	F		T	T	F	**T**	T			**T**			**T**
T	F	T		T	T	T	**F**	F			**F**			**F**

T	F	F	T	T	T	**T**	T	**F**	**T**
F	T	T	F	F	F	**T**	F	**F**	**F**
F	T	F	F	F	F	**T**	T	**F**	**T**
F	F	T	F	T	T	**F**	F	**T**	**F**
F	F	F	F	T	T	**T**	T	**T**	**T**

No row with all true premises and a false conclusion.

IV.

6. Invalid

1. (W • S) ⊃ ~V
2 ~W /∴ S ⊃ V

						p1		p2		c		
W	S	V	(W	•	S)	⊃	~V	~W	S	⊃	V	
T	T	T	T	T	T	**F**	F	**F**		**T**		
T	T	F	T	T	T	**T**	T	**F**		**F**		
T	F	T	T	F	F	**T**	F	**F**		**T**		
T	F	F	T	F	F	**T**	T	**F**		**T**		
F	T	T	F	F	T	**T**	F	**T**		**T**		
F	T	F	F	F	T	***T***	T	***T***		***F***		***
F	F	T	F	F	F	**T**	F	**T**		**T**		
F	F	F	F	F	F	**T**	T	**T**		**T**		

One row (6th) with all true premises and a false conclusion.

V.

4.

p ⊃ q
q ⊃ r /∴ p ⊃ r

Why is it unnecessary to map ~(S G) and ~P as ~p and ~r respectively?

7.

p • q
/∴ p

It is possible for arguments to have only one premise.

VI.

3.

T	(A	≡	B)	⊃	C					A = T
	T		F		F					B = F
		F			F					C = F
T	B	⊃	A							
	F		T							
T	~C				/∴	~A	v	B		
						~T	v	F		
						F	v	F		
							F		Invalid	